جامعة الفرات الاوسط التقنية
كلية البوليتكنك كربلاء
قسم هندسة التقنيات الميكانيكية

# ملزمة الميكانيك الهندسي

# المرحلة الأولى

# Dynamics

# قسم التقنيات الميكانيكية

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مدرس المادة

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## الداينميك : ( Dynamics )

**الداينميك ( علم الحركة ) وهو العلم الذي يتناول حركة الأجسام والقوة المؤثرة عليها .**

**Dynamics**

**kinetics**

**هو العلم الذي يدرس العلاقة بين القوة المؤثرة على الجسم وحركة الجسم**

**Kinematics**

**هو العلم الذي يدرس حركة الحبيبة والاجسام بدون اعتبار القوة المسببة للحركة (يعني يعالج العلاقة بين الازاحة، السرعة، التعجيل)**

**تاريخ علم الديناميكا(Dynamics)**

يُنسب بداية الفهم العلمي المنطقي لعلم الديناميكا إلى غاليليو(1564–1642) ، الذي أجرى ملاحظات حول السقوط الحر، والحركة على سطح مائل، وحركة البندول.

أما نيوتن(1642–1727) ، فقد استرشد بأعمال غاليليو وصاغ قوانين الحركة وقانون الجاذبية الكونية .وقد نُشر عمله الشهير في الطبعة الأولى من كتاب برينسيبيا .(Principia) كما قدم علماء آخرون مثل أويلر (Euler)ودالمبير (D'Alembert) ولاغرانج (Lagrange) إسهامات مهمة في علم الميكانيك.

**الديناميكا(Dynamics)**

هي فرع من علم الميكانيك الذي يدرس حركة الأجسام تحت تأثير القوى.

وتنقسم الديناميكا إلى قسمين:

1. **الكينماتيكا(Kinematics)**
2. **الكينتيكا(Kinetics)**

**الكينماتيكا(Kinematics)**

هي دراسة الحركة دون النظر إلى القوى التي تسبب هذه الحركة.

**الكينتيكا(Kinetics)**

هي دراسة تأثير القوى على الأجسام والحركة الناتجة عنها.

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**تطبيقات الديناميكا**

1. تحليل وتصميم الهياكل المتحركة.
2. الهياكل الثابتة المعرضة لأحمال الصدمات.
3. الأجهزة الروبوتية.
4. تشغيل وتصنيع التوربينات والمضخات.
5. أنظمة التحكم الآلي.
6. الصواريخ.
7. القذائف والمركبات الفضائية.
8. وسائل النقل البرية والجوية وغيرها.

**المفاهيم الأساسية**

الفضاء:(Space)

هو الحيز الهندسي الذي تشغله الأجسام.

**الزمن:(Time)**

هو مقياس تعاقب الأحداث (ويُعتبر مطلقًا في ميكانيكا نيوتن).

**الكتلة:(Mass)**

هي المقياس الكمي للقصور الذاتي أو مقاومة الجسم لتغيير حالته الحركية.

**القوة:(Force)**

هي تأثير متجهي يمارسه جسم على جسم آخر.

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**الجسيم(Particle)**

هو جسم أبعاده مهملة.

يمكن اعتبار الجسم جسيمًا عندما تكون أبعاده غير مهمة في وصف حركته أو تأثير القوى عليه.

**الجسم الصلب(Rigid Body)**

هو جسم تكون تغيرات شكله مهملة مقارنة بأبعاده الكلية أو مقارنة بتغير موقعه ككل.

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**الكميات المتجهة والقياسية(Vector and Scalar)**

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**قوانين نيوتن**

**القانون الأول**

يبقى الجسيم ساكنًا أو يستمر في الحركة بسرعة منتظمة في خط مستقيم إذا لم تؤثر عليه قوة محصلة غير متزنة.

**القانون الثاني**

يتناسب تسارع الجسيم طرديًا مع القوة المحصلة المؤثرة عليه ويكون في اتجاهها.

$$\vec{F} = m\vec{a}$$

**القانون الثالث**

قوى الفعل ورد الفعل بين الأجسام المتفاعلة متساوية في المقدار، ومتعاكسة في الاتجاه، وتقع على نفس خط التأثير.

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**الوحدات(Units)**

تُعرض الوحدات والرموز للكميات الأساسية الأربع في الميكانيك في كل من:

- **النظام الدولي للوحدات(SI)**
- **النظام الأمريكي(U.S. customary system)**

وذلك في الجدول التالي.

| Quantity | Symbol | SI units | Unit Symbol | U.S. unit | Customary unit symbol |
|---|---|---|---|---|---|
| Mass | M | kilogram | kg | Slug | - |
| Length | L | Meter | m | foot | ft |
| Time | T | Second | s | second | s |
| Force | F | Newton | N | pound | lb |

**القيمة القياسية للتسارع الأرضي(g)**

القيمة القياسية التي تم اعتمادها دوليًا لتسارع الجاذبية بالنسبة للأرض الدوّارة عند مستوى سطح البحر وعند خط عرض 45 °هي:

9.80665متر/ثانية$^2$

أو

32.1740قدم/ثانية$^2$

**وعمليًا تُستخدم القيم التقريبية التالية:**

- 9.81متر/ثانية$^2$ في النظام الدولي للوحدات.(SI)
- 32.2قدم/ثانية$^2$ في النظام الأمريكي للوحدات.

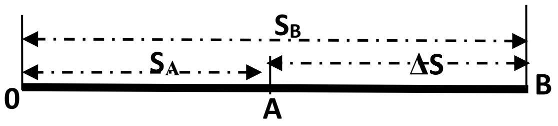
وهذه القيم تمثل قيمة تسارع الجاذبية عند مستوى سطح البحر

**1 – Kinematics**

**1. Rectilinear motion . على خط مستقيم**

**2. Curvilinear motion . على خط منحني**

**3. Rotation . الحرآة الدورانية**




**<u>الحركة الخطية :</u> ( Rectilinear motion )**

**وهي حركة الجسيم على خط مستقيم**

**<u>ملاحظة</u>** : هنا في هذا المسار الخطي فان التغيير في الازاحة ΔS يساوي عدديا ً المسافة بين AB

$S_A$: 0 Displacement (الازاحة A ) موقع الحبيبة ( الجسيم ) في نقطة A بالنسبة للنقطة

$S_B$: 0 Displacement **(** الازاحة B ) موقع الحبيبة في نقطة B مقاسة بالنسبة للنقطة

**التغيير في الازاحة** $\Delta S = S_B - S_A$

الازاحة (S):-

The change of the location of the body with respect to fixed point

تغيير موقع الجسم بالنسبة لنقطة ثابتة

**السرعة الخطية V : هي معدل تغيير الازاحة بالنسبة للزمن .**

$$V_{av} = \frac{\Delta s}{\Delta t}$$

**وعليه السرعة اللحظية ( Instantaneous velocity )**

$$V = \lim_{\Delta t \to 0} it \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \text{..................... ......}(1)$$

**التعجيل الخطيa :** يعرف على انه معدل التغيير في السرعة بالنسبة للزمن

$$a_{av} = \frac{\Delta v}{\Delta t}$$

**وعليه فأن التعجيل اللحظي( a) يساوي**

$$a = \lim_{\Delta t \to 0} it \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \text{..................... ......}(2)$$

**and**

$$a = \frac{d^2 s}{dt^2}$$

**من معادلة رقم(1)**

$$V = \frac{ds}{dt} \Rightarrow dt = \frac{ds}{V}$$

**نعوض في المعادلة رقم ( 2 ) نحصل**

$$a = \frac{dv}{dt} = \frac{dv}{\frac{ds}{v}}$$

$$a = v \frac{dv}{ds} \text{.......... ....}(3)$$

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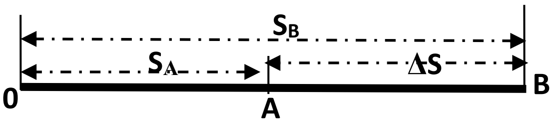
**<u>الحركة الخطية :</u> ( Rectilinear motion )**

**وهي حركة الجسيم على خط مستقيم**




**<u>ملاحظة</u>** : هنا في هذا المسار الخطي فان التغيير في الازاحة $\Delta S$ يساوي عدديا ً المسافة بين AB

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$S_B$: 0 Displacement ( الازاحة B ) موقع الحبيبة في نقطة B مقاسة بالنسبة للنقطة

**$\Delta S = S_B - S_A$ التغيير في الازاحة**

الازاحة (S):-

The change of the location of the body with respect to fixed point

تغيير موقع الجسم بالنسبة لنقطة ثابتة

## Newton's laws قوانين نيوتن

### القانون الاول

**When the Resultant of force acting on a body is zero the body either at rest or moving with constant velocity on straight line عندما تكون محصلة القوى المسلطة على الجسم صفراً فأن الجسم أما ان يبقى ساكناً او يتحرك على خط مستقيم وبسرعة ثابتة .**

V = constant        a = 0

$V = \frac{s}{t}$        v = السرعة     s = الازاحة        t = الزمن

### **Second law     القانون الثاني**

**If the resultant of force acting on body is not zero when the body move in the direction of resultant with acceleration directly proportional to the resultant and inversely proportional to the mass of body .**

$a \; \alpha \; \frac{R}{m}$

a = التعجيل
R= المحصلة
m= الكتلة

اذا كانت محصلة القوى المسلطة على الجسم لاتساوي صفر فأن الجسم سوف يتحربأتجاه المحصلة بتعجيل يتناسب طردياً مـ ع المحصلة وعكسياً مع كتلة الجسم .

## Third law القانون الثالث

**For each action are reactions equal in magnitude and opposite in direction. لكل فعل رد فعل يساويه في المقدار ويعاكسه في الاتجاه.**

**Rectilinear motion with constant acceleration الحركة على خط مستقيم وبتعجيل ثابت**

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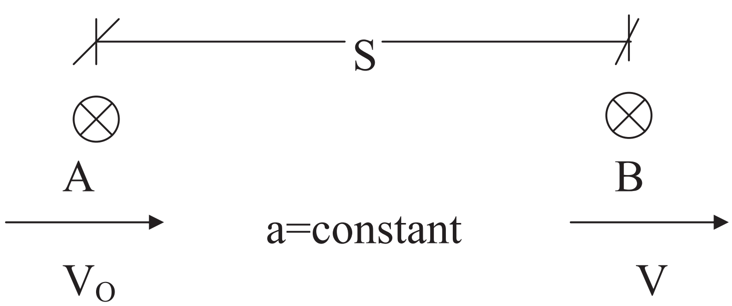
**Let the body moves with constant acceleration (a) from point (A) to point (B) which (s) displacement from (A) in a time (t) starting with initial velocity ($V_O$) at (A) and reach a final velocity (V) at (B) .**




**اذا كان الجسم يتحرك بتعجيل ثابت (a) من النقطة (A) الى النقطة (B) حيث انه قطع ازاحة مقدارها (s) بزمن مقداره (t) مبتدأ بسرعة ابتدائية مقدارها ($V_O$) من النقطة (A) ومنتهياً بسرعة مقدارها (V) في النقطة (B) .**

$$V = \frac{ds}{dt} \quad \text{———} \quad (1)$$

$$a = \frac{dv}{dt} \quad \text{———} \quad (2)$$

From 1 and 2

$$dt = \frac{ds}{v} \qquad dt = \frac{dv}{a}$$

$$\frac{ds}{v} = \frac{dv}{a}$$

Vdv = ads ———— ③

From 2

dv=adt

$$\int_{v_o}^{v} dv = \int_{o}^{t} dt$$

$$V \Big]_{v_o}^{v} = a\left[t\right]_{o}^{t}$$

$( V - V_o ) = a ( t – o )$

$V = V_O + at$ ———— Ⓐ

From ①

ds = Vdt

$$\int_{o}^{s} ds = \int_{o}^{t} (vo + at)\, dt$$

$$\left[s\right]_{o}^{s} = \left[v_o t + \frac{at^2}{2}\right]_{o}^{t}$$

$$(s – o) = v_o t + \frac{1}{2} at^2$$

$S = V_o t + \frac{1}{2} at^2$ ———— Ⓑ

From ③

$Vdv = ads$

$$\int_{vo}^{v} V\,dv = a\int_{o}^{s} ds$$

$$\frac{V^2}{2}\Big]_{VO}^{V} = a\,[s]_{o}^{s}$$

$$\frac{V^2}{2} - \frac{V_o^2}{2} = a(s - o)$$

$$\frac{V^2 - V_o^2}{2} = as$$

$$V^2 - V_o^2 = 2as \qquad \boxed{V^2 = V_O^2 + 2as} \;\text{———}\; (c)$$

a ⟶ التعجيل $(m/sec^2)$

t ⟶ الزمن (sec)

$V_o$ ⟶ السرعة الابتدائية (m/sec)

V ⟶ السرعة النهائية (m/sec)

S ⟶ **الازاحة (m)**

**الخلاصة:**

| **المعادلات المستخدمة في حالة الحركات الخطية** | **المعادلات المستخدمة للحركة الخطية بتعجيل ثابت** |
|---|---|
| **$v = ds/dt$** | **$v = v_o + a * t$** |
| **$a = dv/dt = d^2s/dt^2$** | **$S = s_o + v_o t + 1/2\ at^2$** |
| **$vdv = ads$** | **$v^2 = v_o^2 + 2a(S - S_o)$** |

3- للحركة المنتظمة Uniform motion فأن

ا-معدل السرعة $v_{av}$ = السرعة بعد منتصف الوقت الكلي للحركة

ب-المسافة التي يقطعها الجسم عند اي ثانية = السرعة عند نصف تلك الثانية

4-إذا سقط الجسم من السكون ايضاً $v_o = o$

5-إذا قذف الجسم الى الاعلى فان السرعة عند اقصى ارتفاع يصله الجسم تساوي صفر

**$Ex_1$: A particle moves in Rectilinear motion according to the relation as $S=t^3-9t^2-2$. Determine the displacement(s), the velocity (v) and acceleration (a) when t= 5sec.**

<u>Sol:</u>

$S=t^3-9t^2-2$

At t=5sec

$\therefore S=(5)^3-9(5)^2-2=102$ m

نجد مشتقة الإزاحة لنحصل على السرعة

$$v=\frac{ds}{dt}=3t^2-18t \qquad \text{at } t=5sec$$

$\therefore V=3(5)^2-18(5)=-15$m/sec

نجد مشتقة السرعة لنحصل على التعجيل

**a = dv/dt at t=5sec**

$\therefore a=6(5)-18=12$m/sec$^2$

**<u>$Ex_2$</u>: A body move in Rectilinear motion with acceleration of ($a=-2$m/sec$^2$) if the velocity = 8m/sec and S=0 when t= 0. Determine V & S when t = 6 sec.**

**<u>Sol:</u>**

a = dv/dt

$\therefore dv=a.dt$

بأخذ التكامل للطرفين

$\int dv=\int adt$

**V=** $\int -2dt$

$V=-2t+C_1$ at $t=0\rightarrow v=8$m/sec

$\therefore 8=-2(0)+C_1$ $\therefore C_1=8$

| V=-2t + 8 |
|---|

نجد السرعة عند الزمن 6sec

at t =6 sec $\therefore v=-2(6)+8)$

$\therefore v=-4$m /sec

$$v = \frac{ds}{dt}$$

ds=v.dt

$\therefore$ ds = ( -2t +8) dt

بأخذ التكامل للطرفين نحصل على

$\int ds = \int(-2t+8)dt$

$\therefore S = \int -2tdt + \int 8dt$

$\therefore S = -2t^2/2 + 8t + C_2$

$S = -t^2 + 8t + C_2$

لكن هو معطي في السؤال S=0 at t=0 نعوض في العلاقة أعلاه

$0 = 0+0+C_2 \quad \rightarrow \quad C_2 = 0$

$S = -t^2 + 8t$

t=6sec نجد الازاحة عندما يكون

$S = -(6)^2 + 8(6) \quad = -36 + 48 = 12$ m

**أمثلة خاصة بالحركة الخطية بتعجيل ثابت**

**Ex$_1$: A body started from rest with constant acceleration of 4 m/sec$^2$.**

**1-Find the velocity and displacement of the body after 10sec from its motion.**

**2-Find the distance that the body was traveled when its velocity becomes 12 m/sec.**

**Sol** : نستخدم المعادلات الخاصة بحاله الحركة الخطية بتعجيل ثابت

$V = v_o + a.t$ at $v_o = 0$ , $a = 4 m/sec^2$

**1-** $V = 0 + 4(10) \rightarrow V = 40 m/sec$


$$\boxed{s = s_0 + v_0 t + \frac{1}{2}at^2}$$

$$\therefore s = 0 + 0 + \frac{1}{2}(4)(10)^2 = 200m$$

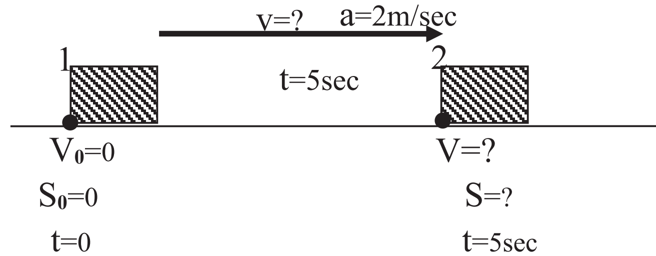
المسافة التي يقطعها الجسم بعد 10 ثواني

**2-** $V^2 = Vo^2 + 2a(S - So)$

$(12)^2 = 0 + 2(4)(S-0) \rightarrow 144 = 8(S)$

$S = \frac{144}{8} = 18m$ 12m/s المسافة التي يقطعها الجسم عندما السرعة

**Ex2: The body in fig below has a constant acceleration 2m/sec if its start from rest , Determine its velocity and position when t = 5 sec .**




**Sol :**

$V_o = 0$

$V = V_o + a.t$ at t=5sec

$V = 0+2*(5) = 10$ m/sec

$$S = S_0 + v_0 t + \frac{1}{2}at^2 \Rightarrow S = 0 + 0 + \frac{1}{2} \times 2 \times 5^2 = 25m$$

**مثال على السقوط الحر**

**Ex3: A stone is thrown vertically upward from the ground with a velocity of 19.6 m/sec.**

1-Find the velocity of the stone after t= 1.5 sec from motion.

2-Find the time which stone needs to reach the highest elevation.

3-Find the time which the stone needs to reach the height of 18.37m.

**Sol:**

**1-** $V_0$=19.6m/sec & g=9.8m/sec$^2$

$V = V_o - gt$

$V = 19.6 - 9.8\,(1.5) = 4.9$ m/sec

**2-**

$V = V_o - g.t$

$V_B = V_A - gt$ , $V_B = 0$ (لأنه أعلى أرتفاع تصل فيه السرعةالى صفر)

$0 = 19.6 - 9.8(t)$

$\therefore 9.8(t) = 19.6$ $\Rightarrow t = \frac{19.6}{9.8} = 2\sec$

$$S = S_0 + v_0 t + \frac{1}{2}gt^2$$

$$18.37 = 0 + 19.6(t) - \frac{1}{2}(9.8)t^2$$

$18.37 = 19.6\,t - 4.9t^2$ نعيد كتابة المعادلة بصورة اخرى لنحصل على

$4.9t^2 - 19.6t + 18.37 = 0$

نضرب المعادلة × 2 نحصل على

$9.8t^2 - 39.2t + 36.75 = 0$

From ③

$Vdv = ads$

$$\int_{vo}^{v} V\,dv = a\int_{o}^{s} ds$$

$$\frac{V^2}{2}\Big]_{VO}^{V} = a\,[s]_o^s$$

$$\frac{V^2}{2} - \frac{V_o^2}{2} = a(s - o)$$

$$\frac{V^2 - V_o^2}{2} = as$$

$$V^2 - V_o^2 = 2as$$

$$\boxed{V^2 = V_O^2 + 2as} \quad \text{———} \quad ⓒ$$

a ⟶ التعجيل (m/sec$^2$)

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V ⟶ السرعة النهائية (m/sec)

S ⟶ **الازاحة (m)**

## Freely fulling bodies سقوط الاجسام سقوطاً حراً

**يعتبر سقوط الاجسام سقوطاً حراً حركة على خط مستقيم وبتعجيل ثابت (التعجيل الارضي)الذي يرمز له (g) والذي يساوي (9.81 m/sec$^2$) لذا فأن قوانين الحركة تصبح على النحو التالي :**

$V = V_o + gt$

$S = V_o t + \frac{1}{2} gt^2$

$V^2 = V_o^2 + 2gs$

**ملاحظة** يعوض عن التعجيل التسارعي (+) والتعجيل التباطؤي (-) في حالة تطبيق قوانين الحركة .

*** في حالة السقوط الحر في حالة سقوط الجسم من الاعلى الى اسفل يكون (g) موجباً (+) وفي حالة قذف الجسم الى الاعلى يكون (g) سالباً (-) .**

**حركة الجسم**

**a= constant تعجيل ثابت** | **V=constant سرعة ثابتة**

$V = V_O + at$

$S = V_o t + \frac{1}{2} a t^2$

$V^2 = V_o^2 + 2as$

$V = \frac{s}{t}$

$a = 0$

**التعجيل**

**تسارع (+)** — **تباطؤ (-)**

**هنالك جملة ملاحظات مهمة حول هذا الموضوع:**

1-عندما يبدا الجسم بالحركة من السكون فان t=o and vo = o

2-عند توقف الجسم عن الحركة فان v = o السرعة النهائية

3- للحركة المنتظمة Uniform motion فأن

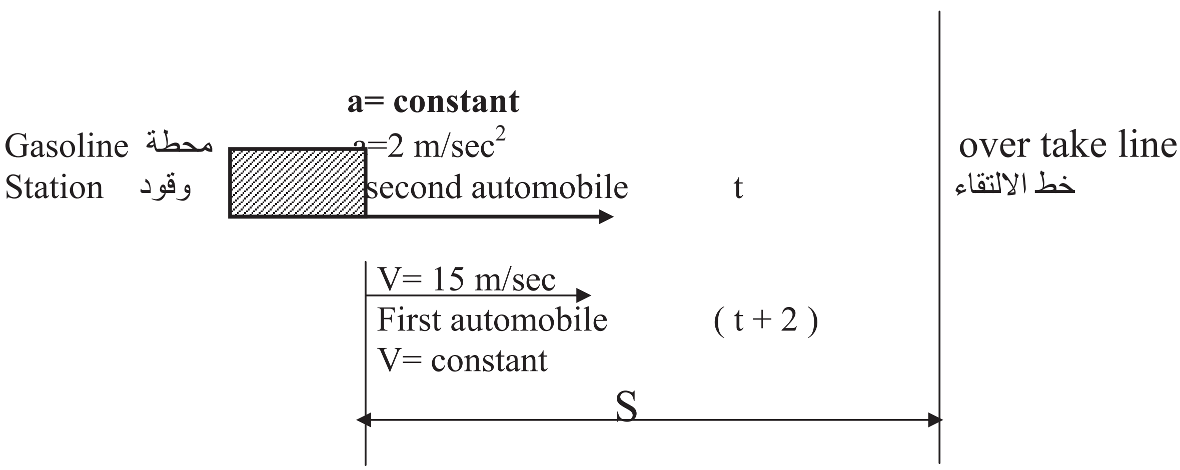
ا-معدل السرعة $v_{av}$ = السرعة بعد منتصف الوقت الكلي للحركة

ب-المسافة التي يقطعها الجسم عند اي ثانية = السرعة عند نصف تلك الثانية

4-إذا سقط الجسم من السكون ايضاً $v_o = o$

5-إذا قذف الجسم الى الاعلى فان السرعة عند اقصى ارتفاع يصله الجسم تساوي صفر

**Q/ An automobile سيارة moving at constant velocity سرعتها ثابتة of ( 15 m/sec) passes a gasoline station محطة وقود two second later بعد ثانيتين , another automobile سيارة اخرى leaves غادرة the gasoline station محطة الوقود accelerated at the constant rate وتسارعت بتعجيل ثابت of ($2 m/sec^2$) , How soon will the second automobile over take the first . ماهو الزمن اللازم للثانية لتصل الى الاولى**



## Second automobile السيارة الثانية

a = constant = 2 m/sec$^2$

$S = V_o t + \frac{1}{2} a t^2$

$S = 0 + \frac{1}{2} * 2 * t^2$

$S = t^2$ —— (1)

## First automobile السيارة الاولى

V = constant = 15 m/ sec

$V = \frac{s}{t}$

$15 = \frac{s}{(t+2)}$

$S = 15t + 30$ —— 2

From 1 and 2

$t^2 = 15t + 30$

$t^2 - 15t - 30 = 0$

$Ax^2 + Bx + c = \mathbf{0}$

$$\mathbf{X} = \frac{-B \pm \sqrt{B^2 - 4Ac}}{2A}$$

$$t = \frac{-(-15) \pm \sqrt{(-15)^2 - 4*1*30}}{2}$$

$$t = \frac{15 \pm \sqrt{345}}{2} = \frac{15 \pm 18.5}{2}$$

$$t = \frac{15 + 18.5}{2} = \frac{33.5}{2}$$

**t = 16.72 sec**

**Q/ A stone is dropped down the well and (5 sec ) later the sound of the splash صوت الاصطدام is heard .**
**If the velocity of sound is (340 m/sec) , what is the depth of the well ?**

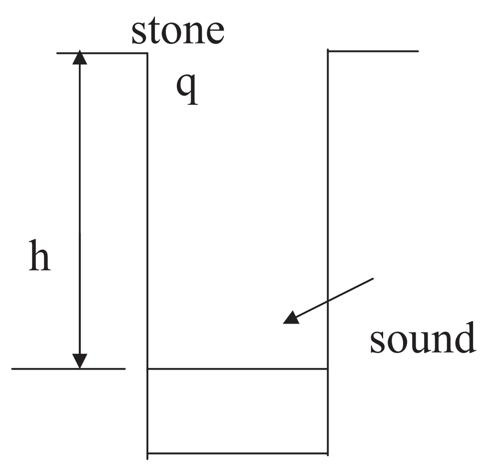
h = نفرض عمق البئر

5 ثانية = زمن نزول الحجر + زمن صعود الصوت

t = نفرض زمن نزول الحجر

5 – t = زمن صعود الصوت

**Foe the stone بالنسبة للحجر**




a = constant = g = 9.81 m/sec$^2$

$S = V_o t + \frac{1}{2} g t^2$

$h = 0 + \frac{1}{2} * 9.81 * t^2$

$h = 4.905\, t^2$ ———— 1

**For the sound بالنسبة للصوت**

V = constant = 340 m/sec

$V = \frac{s}{t}$

$340 = \frac{h}{5-t}$

340*5-340t = h

1700 – 340t = h ———— 2

h = 1700 – 340 t

من 1 , 2

$4.905\, t^2 = 1700 - 340\, t$

$4.905\, t^2 + 340\, t - 1700 = 0$

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$t = \frac{-340 \pm \sqrt{(340)^2 - 4*4.905*-1700}}{2*4.905}$$

t = 4.683 sec

نعوض في 1

h = 4.905 * 4.683

h = 107 m

## 16 th week الاسبوع السادس عشر

## Kinetics of rectilinear translation with constant acceleration

## الحركة على خط مستقيم وبتعجيل ثابت وتحت تأثير قوة

**In rectilinear motion في الحركة على خط مستقيم all part of the body جميع اجزاء الجسم moves in direction بأتجاه parallel to the line of motion يوازي خط الحركة and the displacement الازاحة and velocity السرعة and acceleration to each part لكل جزء from the body are parallel to the line of motion . always x-axis دائماً محور السينات taken positive يؤخذ موجباً in direction of motion بأتجاه الحركة , so the displacement , velocity , acceleration and the components of forces مركبات القوى are positive in the direction of motion بأتجاه الحركة and negative in the opposite direction وسالبة عكس الحركة and if we apply تطبيق second law of Newton قانون نيوتن الثاني so the equation is :**

$$\sum x = \frac{W}{g} * a \qquad \sum Y = 0 \qquad \sum z = 0$$

$\sum x$ = **محصلة القوى بأتجاه الحركة**

W = وزن الجسم

g = التعجيل الارضي

a = تعجيل الجسم

**Ex:** The car moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where $t$ is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.


**Position.** Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates $v$, $s$, and $t$. Noting that $s = 0$ when $t = 0$, we have*

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft} \qquad \textit{Ans.}$$

**Acceleration.** Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates $a$, $v$, and $t$.

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

When $t = 3$ s,

$$a = 6(3) + 2 = 20 \text{ ft/s}^2 \rightarrow$$ *Ans.*

# Freely Falling Bodies

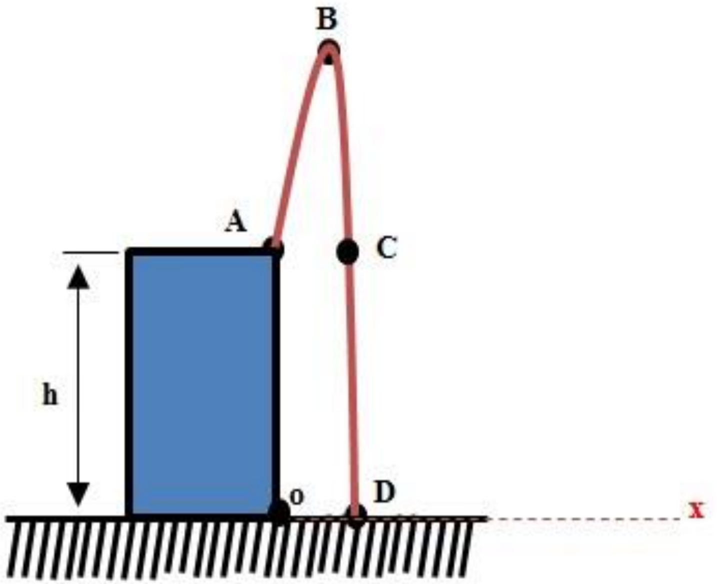
$g = 32.2\ ft/s^2$

$g = 9.81\ m/s^2$

$y_A = h$

$y_D = 0$

$y_c = h$

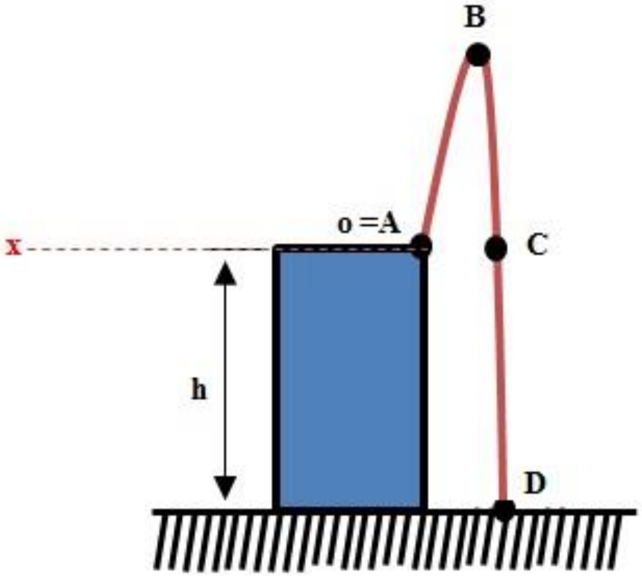



$a = g$ Or $a = -g$

$y_o = y_A = 0$

$y_D = -h$

$y_c = 0$



Ex: a ball is thrown vertically up word from the top of 18 m tower with an initial velocity of 12 m/s . Find?

1) The velocity and displacement at any time.
2) The highest elevation
3) The time when the ball reached the ground and the velocity at that time.

1)

dv/dt = - 9.81

$$\int_{12}^{v} dv = \int_{0}^{t} -9.81\, dt$$

v = - 9.81

v – 12 = - 9.81 t

v = 12 – 9.81 t (Instantaneous velocity)

dy/ dt = 12 – 9.81 t

$$\int_{y_o=0}^{y} dy = \int_{0}^{t} (12 - 9.81t) dt$$

$y = 12\,t - \frac{9.81}{2}\,t^2$ (Instantaneous displacement)

## 2) At highest elevation

$v = 0$

so $0 = 12 - 9.81\,t$ then $t = 1.22$ sec

$y_{at\ 1.22} = 12 * 1.22 - (9.81/2) * (1.22)^2$

$= 7.3$ m

$y = 7.3$ m from the top

$y = 7.3 + 18 = 25.3$ m from the ground

## 3) when the ball hits the ground y = - 18

$-18 = 12\,t - 4.905\,t^2$

$4.905\,t^2 - 12\,t - 18 = 0$

$$t = \frac{12 \mp \sqrt{12^2 + 4 * 18 * 4.905}}{2 * 4.905}$$

Then $t = -10.5$ sec neglected

or $t = 3.5$ sec then $v_{at\ 3.5} = 12 - 9.81 * 3.5 = -22.3$ m/s

EXAMPLE

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3)\ \text{m/s}^2$, where $v$ is in m/s. Determine the projectile's velocity and position 4 s after it is fired.


**Velocity.**

$a = f(v)$ with $v_0 = 60\ \text{m/s}$ when $t = 0$

$(+\downarrow)$

$$a = \frac{dv}{dt} = -0.4v^3$$

$$\int_{60\ \text{m/s}}^{v} \frac{dv}{-0.4v^3} = \int_0^t dt$$

$$\frac{1}{-0.4}\left(\frac{1}{-2}\right)\frac{1}{v^2}\bigg|_{60}^{v} = t - 0$$

$$\frac{1}{0.8}\left[\frac{1}{v^2} - \frac{1}{(60)^2}\right] = t$$

$$v = \left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}\right\}\ \text{m/s}$$

When $t = 4$ s, $v = 0.559\ \text{m/s}\downarrow$ *Ans.*

**Position.**

from $v = ds/dt$ condition $s = 0$, when $t = 0$,

$(+\downarrow)$
$$v = \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}$$

$$\int_0^s ds = \int_0^t \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2} dt$$

$$s = \frac{2}{0.8}\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2}\Bigg|_0^t$$

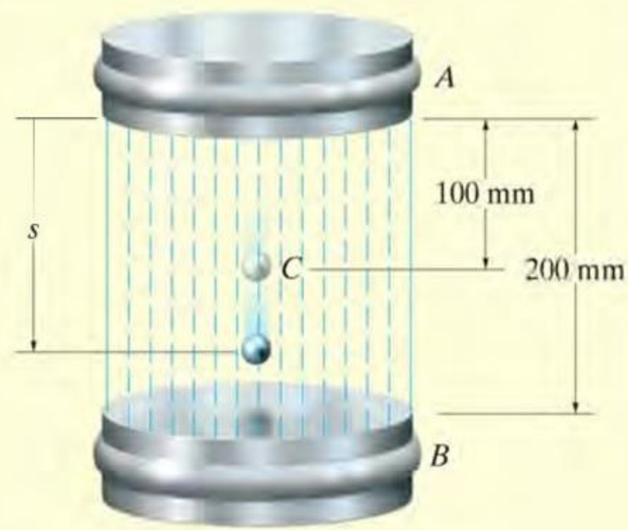
$$s = \frac{1}{0.4}\left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2} - \frac{1}{60}\right\} \text{ m}$$

When $t = 4$ s,

$$s = 4.43 \text{ m}$$ *Ans.*

## EXAMPLE

A metallic particle is subjected to the influence of a magnetic field as it travels downward through a fluid that extends from plate $A$ to plate $B$, Fig. 12–5. If the particle is released from rest at the midpoint $C$, $s = 100$ mm, and the acceleration is $a = (4s)$ m/s$^2$, where $s$ is in meters, determine the velocity of the particle when it reaches plate $B$, $s = 200$ mm, and the time it takes to travel from $C$ to $B$.




## SOLUTION

### Velocity.

Since $a = f(s)$,

$v\,dv = a\,ds$. using $v = 0$ at $s = 0.1$ m

$(+\downarrow)$

$$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_{0.1\text{ m}}^{s} 4s\,ds$$

$$\frac{1}{2}v^2\Big|_0^v = \frac{4}{2}s^2\Big|_{0.1\text{ m}}^{s}$$

$$v = 2(s^2 - 0.01)^{1/2}\text{ m/s}$$

At $s = 200\text{ mm} = 0.2\text{ m}$,

$$v_B = 0.346\text{ m/s} = 346\text{ mm/s}\downarrow \qquad \textit{Ans.}$$

**Time.** The time for the particle to travel from $C$ to $B$ can be obtained using $v = ds/dt$ and Eq. 1, where $s = 0.1$ m when $t = 0$. From Appendix A,

$(+\downarrow)$

$$ds = v\,dt$$

$$= 2(s^2 - 0.01)^{1/2} dt$$

$$\int_{0.1}^{s} \frac{ds}{(s^2 - 0.01)^{1/2}} = \int_0^t 2\,dt$$

$$\ln\left(\sqrt{s^2 - 0.01} + s\right)\Big|_{0.1}^{s} = 2t\Big|_0^t$$

$$\ln\left(\sqrt{s^2 - 0.01} + s\right) + 2.303 = 2t$$

At $s = 0.2$ m,

$$t = \frac{\ln\left(\sqrt{(0.2)^2 - 0.01} + 0.2\right) + 2.303}{2} = 0.658 \text{ s} \qquad \textit{Ans.}$$

a = constant

$$v \frac{dv}{dx} = a$$

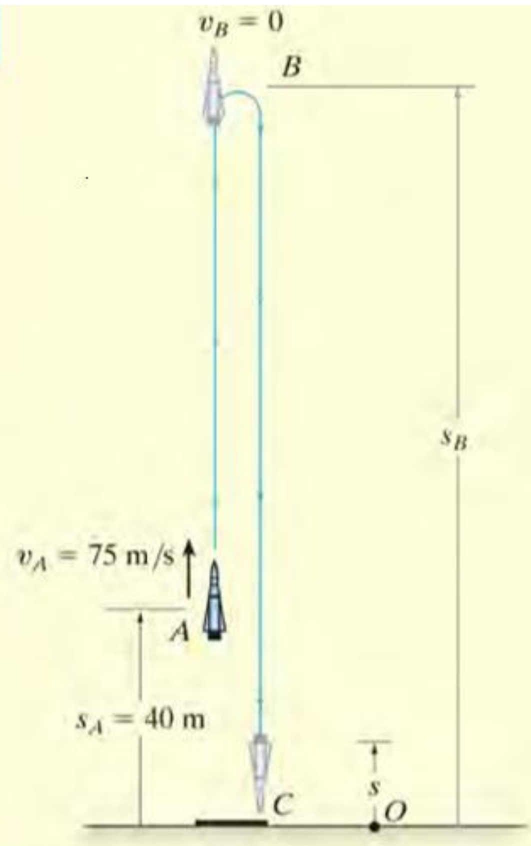
$$\int_{v_o}^{v} v\,dv = \int_{x_o}^{x} a\,dx$$

$$\frac{v^2}{2} = a\,(x - x_o)$$

$$v^2 = v_o^2 + 2\,a\,(x - x_o)$$

## EXAMPLE

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height $s_B$ reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s$^2$ due to gravity. Neglect the effect of air resistance.



## SOLUTION

**Maximum Height.**

$v_B = 0.$ the maximum height $s = s_B$

$v_A = +75\text{m/s}$ when $t = 0$. At

Since $a_c$ is *constant* $a_c = -9.81 \text{ m/s}^2$

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

$$0 = (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(s_B - 40 \text{ m})$$

$$s_B = 327 \text{ m} \qquad \textit{Ans.}$$

**Velocity.** To obtain the velocity of the rocket just before it hits the ground,

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$

$$= 0 + 2(-9.81 \text{ m/s}^2)(0 - 327 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

The negative root was chosen since the rocket is moving downward. Similarly, Eq. 12–6 may also be applied between points $A$ and $C$,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$

$$= (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(0 - 40 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

H. W. :

Chapter 12: 1, 4, 5, 7, 9, 11, 28, 29, 40

**12–1.**

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)\ \text{m/s}^2$, where $t$ is in seconds. What is the particle's velocity when $t = 6$ s, and what is its position when $t = 11$ s?

## SOLUTION

$a = 2t - 6$

$dv = a\,dt$

$$\int_0^v dv = \int_0^t (2t - 6)\,dt$$

$v = t^2 - 6t$

$ds = v\,dt$

$$\int_0^s ds = \int_0^t (t^2 - 6t)\,dt$$

$$s = \frac{t^3}{3} - 3t^2$$

When $t = 6\ s$,

$v = 0$ **Ans.**

When $t = 11\ s$,

$s = 80.7$ m **Ans.**

**Ans:**
$s = 80.7$ m

***12–4.**

A particle travels along a straight line with a constant acceleration. When $s = 4$ ft, $v = 3$ ft/s and when $s = 10$ ft, $v = 8$ ft/s. Determine the velocity as a function of position.

## SOLUTION

***Velocity:*** To determine the constant acceleration $a_c$, set $s_0 = 4$ ft, $v_0 = 3$ ft/s, $s = 10$ ft and $v = 8$ ft/s and apply Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$8^2 = 3^2 + 2a_c\,(10 - 4)$$

$$a_c = 4.583 \text{ ft/s}^2$$

Using the result $a_c = 4.583 \text{ ft/s}^2$, the velocity function can be obtained by applying Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v^2 = 3^2 + 2(4.583)\,(s - 4)$$

$$v = \left(\sqrt{9.17s - 27.7}\right) \text{ft/s} \qquad \textbf{Ans.}$$

**Ans:**
$v = \left(\sqrt{9.17s - 27.7}\right)$ ft/s

**12–5.**

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where $t$ is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

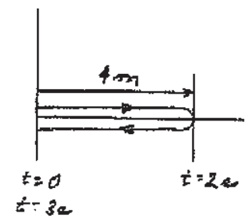
## SOLUTION

$v = 6t - 3t^2$

$a = \dfrac{dv}{dt} = 6 - 6t$

At $t = 3$ s

$a = -12 \text{ m/s}^2$ **Ans.**




$ds = v\,dt$

$$\int_0^s ds = \int_0^t (6t - 3t^2)dt$$

$s = 3t^2 - t^3$

At $t = 3$ s

$s = 0$ **Ans.**

Since $v = 0 = 6t - 3t^2$, when $t = 0$ and $t = 2$ s.

when $t = 2$ s, $s = 3(2)^2 - (2)^3 = 4$ m

$s_T = 4 + 4 = 8$ m **Ans.**

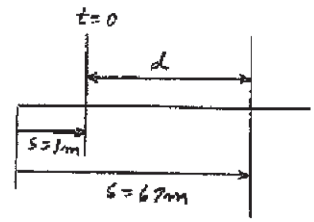
$(v_{sp})_{avg} = \dfrac{s_T}{t} = \dfrac{8}{3} = 2.67$ m/s **Ans.**

**Ans:**
$s_T = 8$ m
$v_{avg} = 2.67$ m/s

**12–9.**

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where $t$ is in seconds. If $s = 1\ \text{m}$ and $v = 2\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6\ \text{s}$. Also, determine the total distance the particle travels during this time period.

## SOLUTION




$a = 2t - 1$

$dv = a\,dt$

$$\int_2^v dv = \int_0^t (2t - 1)dt$$

$v = t^2 - t + 2$

$dx = v\,dt$

$$\int_t^s ds = \int_0^t (t^2 - t + 2)dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$

When $t = 6$ s

$v = 32\ \text{m/s}$ **Ans.**

$s = 67\ \text{m}$ **Ans.**

Since $v \neq 0$ for $0 \leq t \leq 6$ s, then

$d = 67 - 1 = 66\ \text{m}$ **Ans.**

**Ans:**
$v = 32\ \text{m/s}$
$s = 67\ \text{m}$
$d = 66\ \text{m}$

**12–10.**

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})$ m/s$^2$, where $s$ is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

## SOLUTION

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

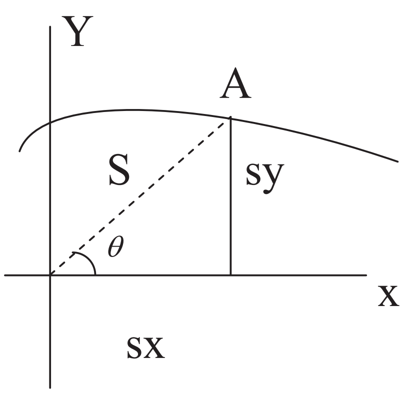
$$0.8351 = \frac{1}{2}v^2$$

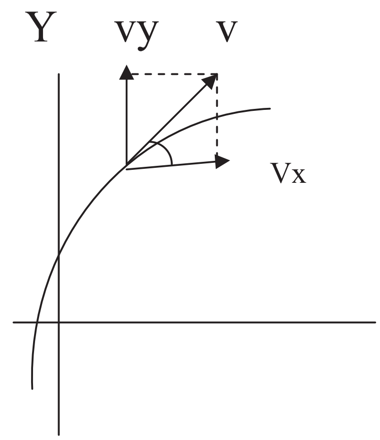
$v = 1.29$ m/s **Ans.**

**Ans:**
$v = 1.29$ m/s

## Curvilinear translations of body حركة الجسم على خط منحني

**In case of moving on straight line في حالة الحركة على خط مستقيم the displace velocity and acceleration are in the direction of motion. in case of curvilinear motion displacement الازاحة velocity السرعة acceleration has two components لها مركبتين . one parallel to x-axis and other parallel to y-axis . Projectile القذائف is sample نموذج of curvilinear motion.**



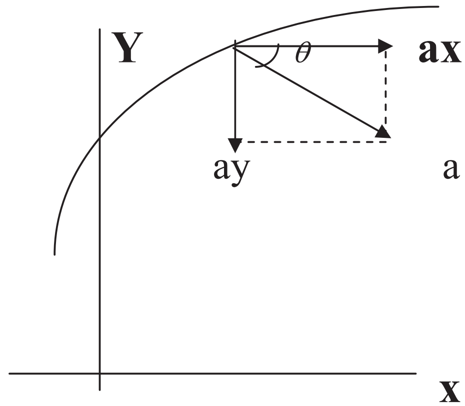

**دائماً تكون السرعة مماس للخط المنحني**

$$V = \sqrt{vx^2 + vy^2}$$

$$\tan\theta = \frac{v_y}{v_x}$$

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$




$$a = \sqrt{ax^2 + ay^2}$$

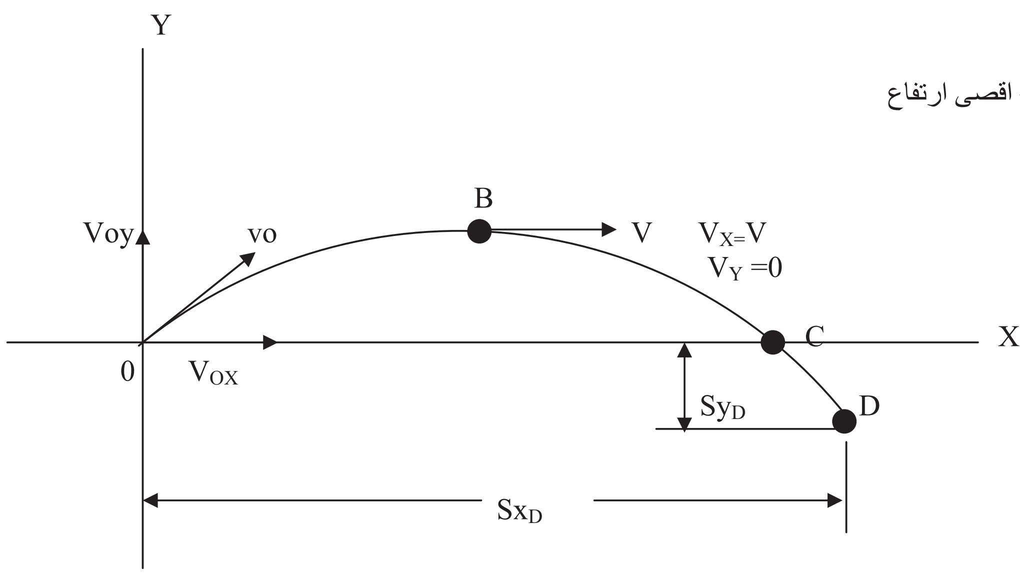
$$\tan\theta = \frac{a_y}{a_x}$$

$$a_x = a\cos\theta$$

$$a_y = a\sin\theta$$

## Flight of projectile Air resistance neglected
## طيران القذائف ، مقاومة الهواء محذوفة

في حالة اقصى ارتفاع



1. Air resistance neglected مقاومة الهواء محذوفة
2. projectile moving in vacuum تتحرك القذيفة في الفراغ
3. wind velocity and rotation of projectile neglected سرعة الريح ودوران القذيفة محذوفة
4. Let the path مسار of projectile القذيفة be given by the curve المنحني ( O , B , C , D ) .
5. let the initial velocity of projectile is ( Vo ) الابتدائية السرعة للقذيفة هي Vo
6. the only force فقط القوة المؤثرة acting on projectile على القذيفة.

Is its weight its total acceleration التعجيل الكلي at all position في جميع المواقع is due to gravity and directed يتجه vertically down ward الى اسفل with value ( $g= 9.81\ m/sec^2$ ) .

$a_x = 0$ $a_y = -g$

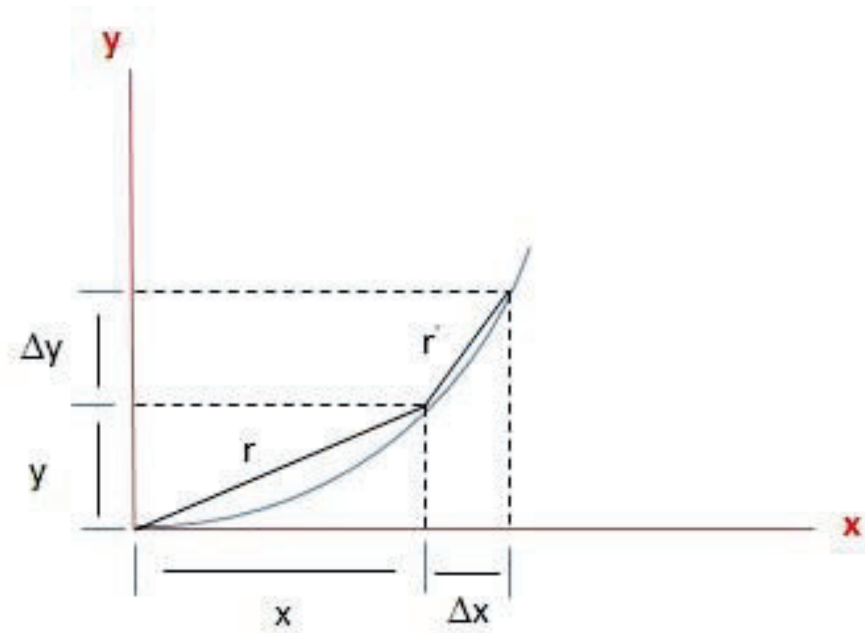
In stead of considering the actual path المسار الحقيقي of projectile we combine its simultaneous projection up on the ( x-axis) and (y-axis) . the equation of these rectilinear components المركبات الخطية of the path are found by substituting تعويض the ( x ) and ( y ) components of (s) , (v) and (a) in the equation معادلات for rectilinear motion with constant acceleration نعوض في معادلات الحركة على خط مستقيم وبتعجيل ثابت.

**As shown كما موضح in a compiling table جدول المقارنة**

| Rectilinear motion constant acceleration | X – component of flight | y- components of flight |
| --- | --- | --- |
| $V = V_O + at$ | $V_X = Vox + axt$<br>$Vx = Vo \cos \theta$ | $Vy = Voy + ayt$<br>$Vy = Vo \sin \theta - gt$ |
| $S = Vot + \frac{1}{2}at^2$ | $Sx = Voxt + \frac{1}{2}axt^2$<br>$Sx = Vo \cos \theta t$ | $Sy = Voyt + \frac{1}{2}ayt^2$<br>$Sy = Vo \sin\theta t - \frac{1}{2}gt^2$ |

# Curvilinear Motion

The motion a long a path other than a straight line is called a curvilinear motion




## Displacement :

In x-direction : x = f(t)

In y-direction : y = g(t)

$r^2 = x^2 + y^2$

$\theta = \tan^{-1} y/x$

$r^{>}$ displacement from o is called the position vector.

## Velocity :

In x-direction

$$v_x = \frac{dx}{dt} = x'$$

In y-direction

$$v_y = \frac{dy}{dt} = y'$$

$$v^> = v_x^> + v_y^> \quad ; \qquad v = \sqrt{v_x^2 + v_y^2}$$

$$\theta = tan^{-1} \frac{v_y}{v_x} = tan^{-1} \frac{dy}{dx}$$

**Acceleration :**

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = x''$$

$$a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2} = y''$$

$$a^{>} = a_x^{>} + a_y^{>} \quad ; \qquad a = \sqrt{a_x^2 + a_y^2}$$

$$\theta = tan^{-1}\frac{d^2y}{d^2x}$$

Ex: A particle is moving along a curve such that $x=(t+1)^2$ and $y=(t+1)^{-2}$ ( x and y in m, t in sec). Find v and a at t = 1 sec.

$V_x = dx/dt = x' = 2(t+1)$

Then $a_x = 2$

$V_y = dy/dt = y' = -2\,(t+1)^{-3}$

Then $a_y = 6\,(t+1)^{-4}$

at t = 1 sec

$v_x = 2(1+1) = 4$ m/s

$v_y = -2\,(1+1)^{-3} = -1/4$ m/s

$v = \sqrt{4^2 + (\frac{-1}{4})^2}$ , v = 4 m/s

$\Theta = \tan^{-1} dy/dx = \tan^{-1} 1/16 = 3.75^o$

$a_x = 2$ m/s$^2$

$a_y = 6\,(1+1)^{-4} = 3/8$ m/s$^2$

$a = \sqrt{2^2 + (\frac{3}{8})^2}$ , v = 2 m/ s$^2$

$\Theta = \tan^{-1} d^2 y/d^2 x = \tan^{-1} 3/16 = 10.62^o$

Ex: If $x = 5t^3$ and $y = 4t^2$ at time $t$, find the magnitude and direction of the velocity when $t = 10$.

$x = 5t^3$ So $dx/dt = 15t^2$

At $t = 10$, the velocity in the $x$-direction is given by:

$dx/dt = v_x = 15(10)^2 = 1500$ m/s

Also, $y = 4t^2$ so the velocity in the $y$-direction is:

$dy/dt = 8t$

When $t = 10$, the velocity in the $y$-direction is:

$dy/dt = v_y = 8(10) = 80$ m/s

So the magnitude of the velocity will be:

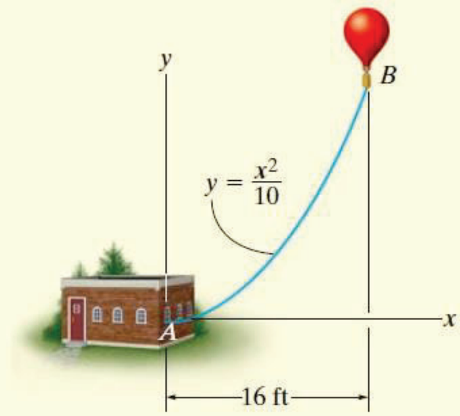
$v = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{1500^2 + 80^2} = 1502.1$ m/s

Now for the direction of the velocity (it is an angle, relative to the positive $x$-axis):

$\tan \theta_v = v_y / v_x$ So $\theta_v = 3.05°$.

## EXAMPLE

At any instant the horizontal position of the weather balloon is defined by $x = (8t)$ ft, where $t$ is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

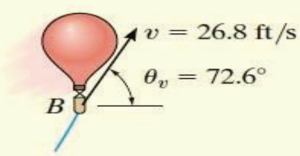



### SOLUTION

**Velocity.** The velocity component in the $x$ direction is

$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ ft/s} \rightarrow$$

$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ ft/s} \uparrow$$



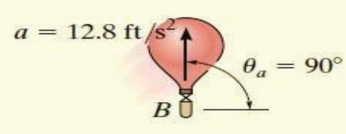
(b)

When $t = 2$ s

$$v = \sqrt{(8 \text{ ft/s})^2 + (25.6 \text{ ft/s})^2} = 26.8 \text{ ft/s} \qquad \textit{Ans.}$$

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6° \qquad \textit{Ans.}$$

**Acceleration.**



(c)

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ ft/s}^2 \uparrow$$

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ ft/s}^2 \qquad \textit{Ans.}$$

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90° \qquad \textit{Ans.}$$

## EXAMPLE


For a short time, the path of the plane is described $y = (0.001x^2)$ m. If the plane is rising with a constant velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it is at $y = 100$ m.

### SOLUTION

When $y = 100$ m, then $100 = 0.001x^2$

$x = 316.2$ m.

$v_y = 10$ m/s,

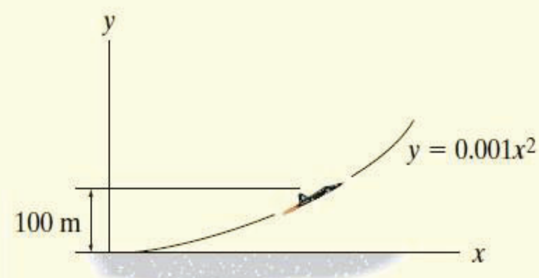
$y = v_y t$; then $100 \text{ m} = (10 \text{ m/s})\, t$ we have $t = 10$ s

**Velocity.**

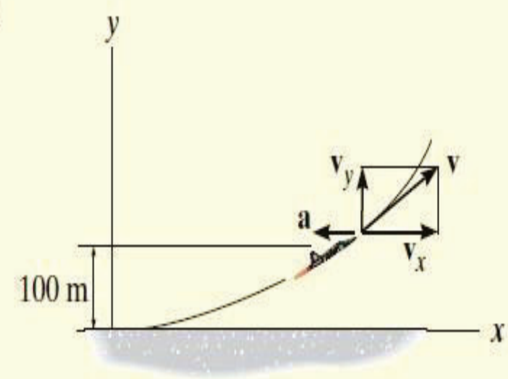
$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x$$

Thus $10 \text{ m/s} = 0.002(316.2 \text{ m})(v_x)$

$v_x = 15.81$ m/s



$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81 \text{ m/s})^2 + (10 \text{ m/s})^2} = 18.7 \text{ m/s} \qquad \textit{Ans}$$




(b)

**Acceleration.**

$$a_y = \dot{v}_y = 0.002\dot{x}v_x + 0.002x\dot{v}_x = 0.002(v_x^2 + xa_x)$$

When $x = 316.2 \text{ m}, v_x = 15.81 \text{ m/s}, \dot{v}_y = a_y = 0,$

$$0 = 0.002((15.81 \text{ m/s})^2 + 316.2 \text{ m}(a_x))$$

$$a_x = -0.791 \text{ m/s}^2$$

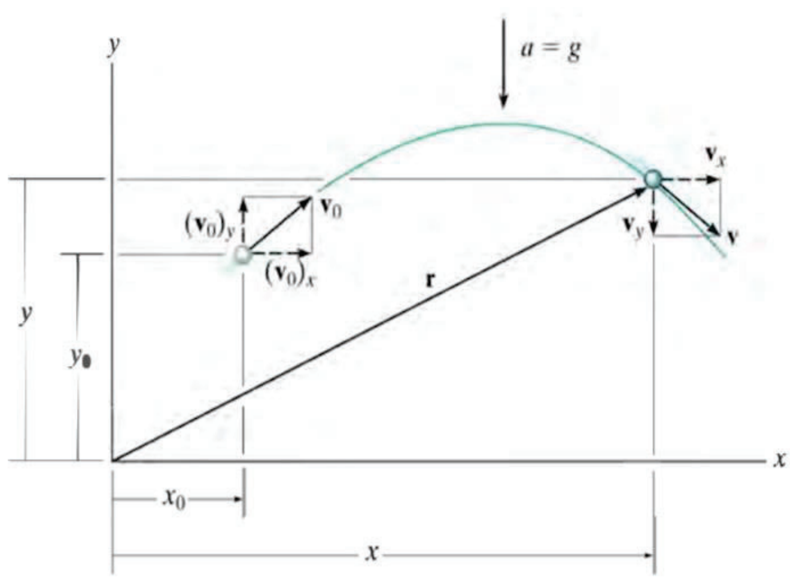
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791 \text{ m/s}^2)^2 + (0 \text{ m/s}^2)^2}$$

$$= 0.791 \text{ m/s}^2 \qquad \textit{Ans.}$$

# Motion of a Projectile

When air resistance is neglected, the only force acting on the projectile is its weight, which causes the projectile to have a constant downward acceleration of:

$$a_c = g = 9.81 \text{ m/s}^2 \quad \text{or} \quad g = 32.2 \text{ ft/s}^2$$



## In x-direction :

Neglecting air resistance $a_x = 0$

$V_x$ = constant = $V_{ox} = V_o \cos \alpha$

$X = X_o + V\,t$

$X = V_o \cos \alpha\, t$ ( when $x_o = 0$ )

## In y-direction :

$a_y = -g$

$V = V_o + a\,t$

$y = y_0 + V_o\, t + ½\, a\, t^2$

$V^2 = V_o^2 + 2\,a\,(y - y_o)$

When the $y_o = 0$ then the equation will be:

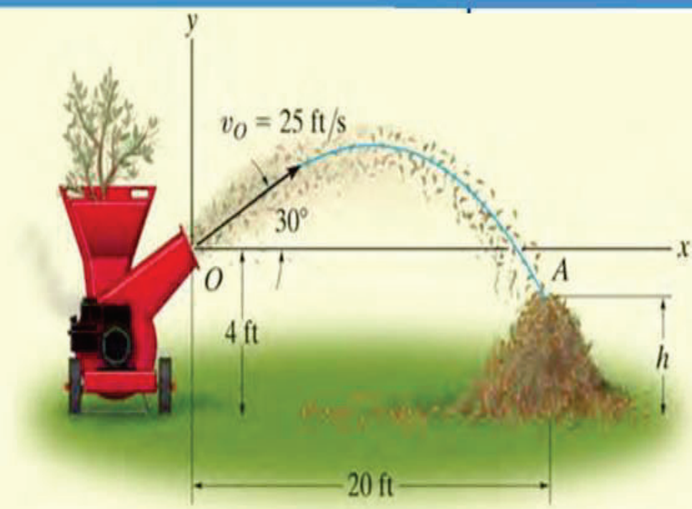
$V_y = V_o \sin \alpha - g\,t$

$y = V_o \sin \alpha\, t - ½\, g\, t^2$

$V_y^2 = (V_o \sin \alpha)^2 - 2\,g\,y$

## EXAMPLE

The chipping machine is designed to eject wood chips at $v_O = 25$ ft/s as shown If the tube is oriented at 30° from the horizontal, determine how high, $h$, the chips strike the pile if at this instant they land on the pile 20 ft from the tube.




X= 20 ft , $\alpha = 30^o$ , $V_o$= 25 ft/s , a = $-32.2$ ft/$s^2$

$X = V_o \cos \alpha\, t$

20 = 25 cos 30 t then t = 0.9238 sec (زمن السقوط)

$y = V_o \sin \alpha\, t - \frac{1}{2} g t^2$

$= 25 \sin 30 * 0.9238 - \frac{1}{2} * 32.2 * 0.9238^2$

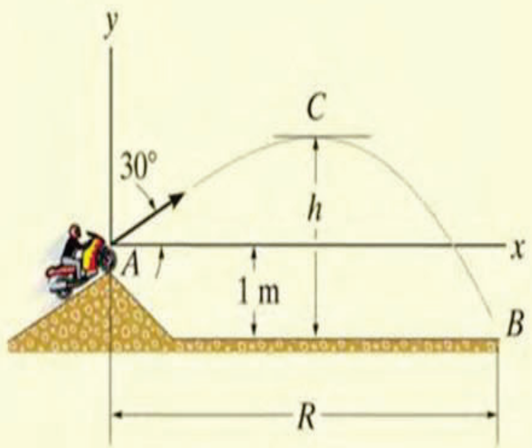
Then y = - 2.19 ft

h = 4 – 2.19

= 1.81 ft

## EXAMPLE

The track for this racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race it was observed that the rider remained in mid air for 1.5 s. Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.




$\alpha = 30^o$ , $y = -1$ m , $t = 1.5$ sec

$x = R = ?$ , $h_{max} = ?$ , $V_y = ?$

$y = V_o \sin \alpha\, t - ½\, g\, t^2$

$-1 = V_o \sin 30 * 1.5 - ½ * 9.81 * 1.5^2$

Then $V_o = 13.382$ m/s

$X = V_o \sin \alpha\, t$

$R = 13.382 \cos 30 * 1.5$

Then $R = 17.384$ m

$y_{max}$ at $V_y = 0$

$V_y^2 = (V_o \sin \alpha)^2 - 2\, g\, y_{max}$

$0 = (13.382 \sin 30)^2 - 2 * 9.81 * y_{max}$

Then $y_{max} = 2.282$ m

$h_{max} = 2.282 + 1$

$= 3.282$ m

Ex: a projectile is shot with an initial velocity of 800 ft/s at a target B located 2000 ft above the gun A and at a horizontal distance 12000 ft . Neglect air resistance, determine firing angle α :

$X = 800 \cos\alpha\, t$

$12000 = 800 \cos\alpha\, t$

$t = 15/\cos\alpha$ ------(1)

$y = 800 \sin\alpha\, t - ½ * 32.2\, t^2$

at point B $2000 = 800 \sin\alpha\, t - 16.1\, t^2$ -----(2)

equ. (1) in (2)

$2000 = 800 \sin\alpha\, (15/\cos\alpha) - 16.1\, (15/\cos\alpha)^2$

$2000 = 800 * 15 \tan\alpha - 16.1 * 15^2 . \sec^2\alpha$

$2000 = 800 * 15 \tan\alpha - 16.1 * 15^2 . (1+ \tan^2\alpha)$

$3622 * \tan^2\alpha - 12000 \tan\alpha + 5622 = 0$

$$\tan\propto = \frac{12000 \mp \sqrt{12000^2 - 4 * 5622 * 3622}}{2 * 3622}$$

Tan α = 1.565 then α = 29.5°

Or

Tan α = 2.75 then α = 70°

## Newton's laws قوانين نيوتن

### القانون الاول

**When the Resultant of force acting on a body is zero the body either at rest or moving with constant velocity on straight line**
**عندما تكون محصلة القوى المسلطة على الجسم صفراً فأن الجسم أمـا ان يبقى سـاكناً او يتحرك على خط مستقيم وبسرعة ثابتة .**

V = constant        a = 0

$V = \frac{s}{t}$        v = السرعة     s = الازاحة        t = الزمن

### Second law      القانون الثاني

**If the resultant of force acting on body is not zero when the body move in the direction of resultant with acceleration directly proportional to the resultant and inversely proportional to the mass of body .**

$$a \;\alpha\; \frac{R}{m}$$

a = التعجيل

R= المحصلة

m= الكتلة

اذا كانت محصلة القوى المسلطة على الجسم لاتساوي صفر فأن الجسم سوف يتحربأتجاه المحصلة بتعجيل يتناسب طردياً مع المحصلة وعكسياً مع كتلة الجسم .

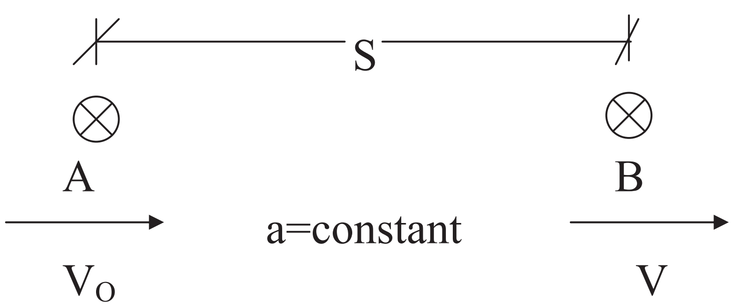
## Third law القانون الثالث

**For each action are reactions equal in magnitude and opposite in direction. لكل فعل رد فعل يساويه في المقدار ويعاكسه في الاتجاه.**

**Rectilinear motion with constant acceleration الحركة على خط مستقيم وبتعجيل ثابت**

**--------------------------------------------------------------------------------**

**Let the body moves with constant acceleration (a) from point (A) to point (B) which (s) displacement from (A) in a time (t) starting with initial velocity ($V_O$) at (A) and reach a final velocity (V) at (B) .**




**اذا كان الجسم يتحرك بتعجيل ثابت (a) من النقطة (A) الى النقطة (B) حيث انه قطع ازاحة مقدارها (s) بزمن مقداره (t) مبتدأ بسرعة ابتدائية مقدارها ($V_O$) من النقطة (A) ومنتهياً بسرعة مقدارها (V) في النقطة (B) .**

$$V = \frac{ds}{dt} \quad \text{———} \quad (1)$$

$$a = \frac{dv}{dt} \quad \text{———} \quad (2)$$

From 1 and 2

$$dt = \frac{ds}{v} \qquad dt = \frac{dv}{a}$$

$$\frac{ds}{v} = \frac{dv}{a}$$

Vdv = ads ———— ③

From 2

dv=adt

$$\int_{v_o}^{v} dv = \int_{o}^{t} dt$$

$$V \Big]_{v_o}^{v} = a\left[t\right]_{o}^{t}$$

( V - $V_o$ ) = a ( t – o )

**V = $V_O$ + at** ———— Ⓐ

From ①

ds = Vdt

$$\int_{o}^{s} ds = \int_{o}^{t} (vo + at)\, dt$$

$$\left[s\right]_{o}^{s} = \left[v_o t + \frac{at^2}{2}\right]_{o}^{t}$$

$$(s - o) = v_o t + \frac{1}{2} at^2$$

**$S = V_o t + \frac{1}{2} at^2$** ———— Ⓑ

مدرس المادة : علاء محمد مرزه

From ③

Vdv = ads

$$\int_{v_o}^{v} V\,dv = a\int_{o}^{s} ds$$

$$\left.\frac{V^2}{2}\right]_{V_O}^{V} = a\,[s]_o^s$$

$$\frac{V^2}{2} - \frac{V_o^2}{2} = a(s - o)$$

$$\frac{V^2 - V_o^2}{2} = as$$

$$V^2 - V_o^2 = 2as$$

$V^2 = V_O^2 + 2as$ —— ⓒ

a → التعجيل ($m/sec^2$)

t → الزمن (sec)

$V_o$ → السرعة الابتدائية (m/sec)

V → السرعة النهائية (m/sec)

S → **الازاحة (m)**

# Energy & work

Definition of Work:

In physics, a force is said to do work if, when acting on a body, there is a displacement of the point of application in the direction of the force.

Units:

The SI unit of work is the joule (J)

N . m = J

1 Ib . ft = 1.356 J

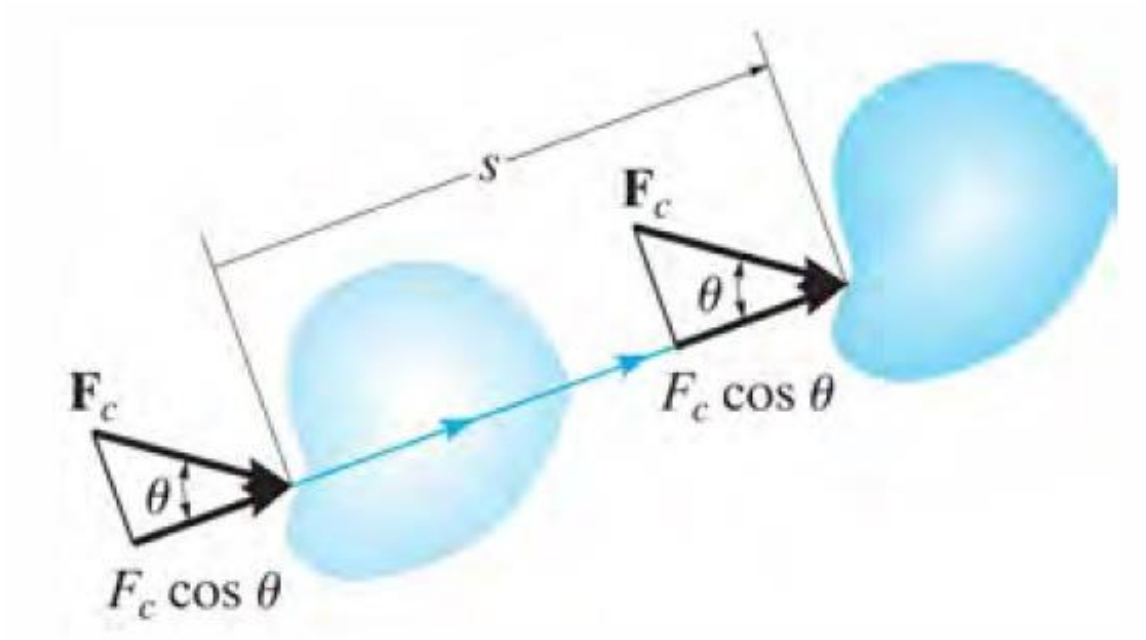
The work **U** done by a constant force of magnitude **F** on a point that moves a displacement **S** is:

$$U = F \cdot S$$

## Work of Constant Force:

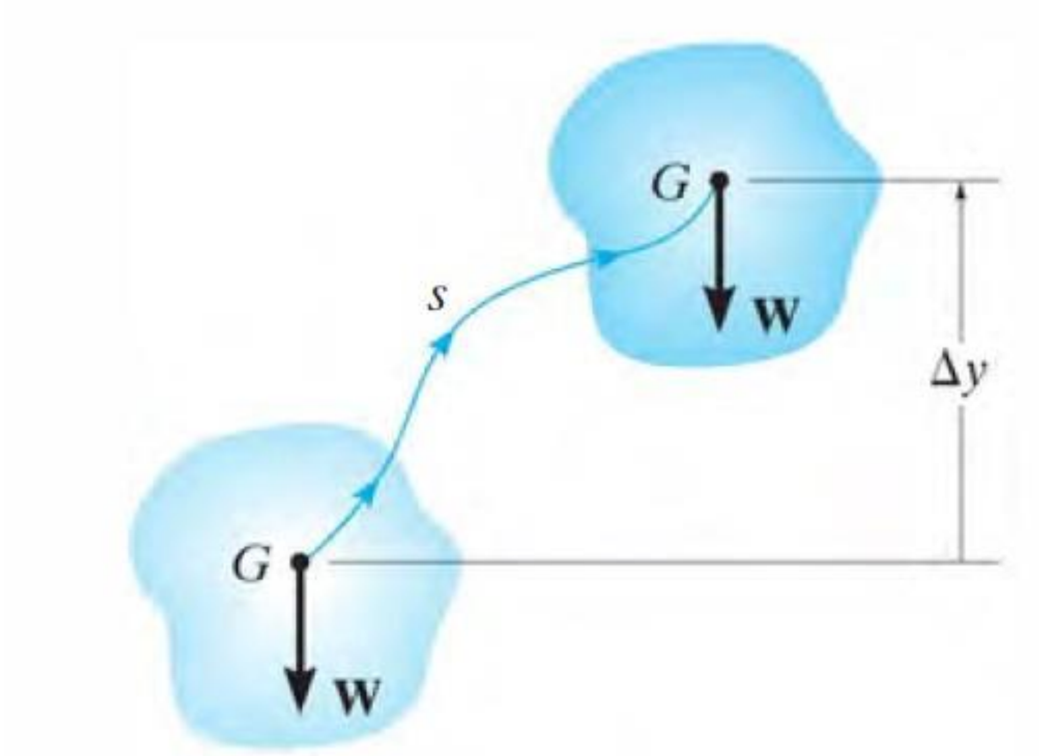
The constant force $F_c$ that acts on the body, which produces a displacement S, can be explained as work equation:

$$U = (F \cos \Theta)\, S$$



# Work of a Weight:

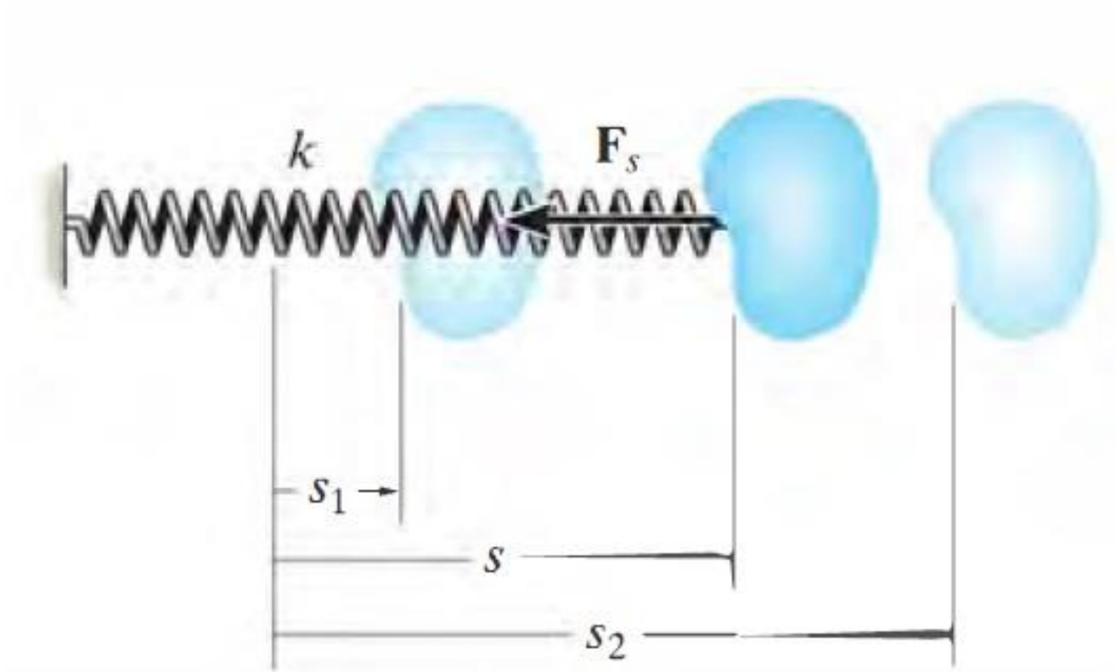
$$\mathbf{U} = \mathbf{W} \cdot \Delta y$$



## Work of a Spring Force:

The spring force is $F = - K . S$

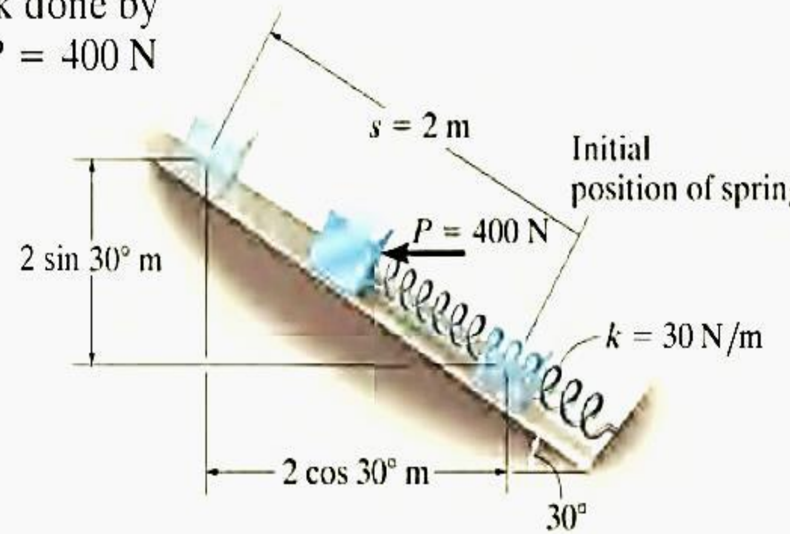
$$U = -\frac{1}{2}(K\,S_2^2 - K\,S_1^2)$$




$$U = -\frac{1}{2}(F_2 - F_1)\Delta s$$

## EXAMPLE

The 10-kg block rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force $P = 400$ N pushes the block up the plane $s = 2$ m.




SOLUTION

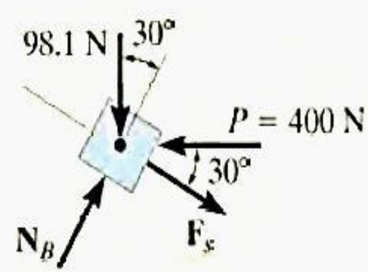
**Horizontal Force *P*.**

Since this force is *constant*, the work is

$$U_P = 400\,(2\cos 30°) = 692.8\text{ J}$$

or

$$U_P = 400\text{ N}\cos 30°(2\text{ m}) = 692.8\text{ J}$$



**Spring Force $F_s$.**

$s_1 = 0.5\ \text{m}$

$s_2 = 0.5\ \text{m} + 2\ \text{m} = 2.5\ \text{m}$

$$U_s = -\left[\tfrac{1}{2}(30\ \text{N/m})(2.5\ \text{m})^2 - \tfrac{1}{2}(30\ \text{N/m})(0.5\ \text{m})^2\right] = -90\ \text{J}$$

**Weight $W$.**

$$U_W = -(98.1\ \text{N})\ (2\ \text{m} \sin 30^\circ) = -98.1\ \text{J}$$

**Total Work.**

$$U_T = 692.8\ \text{J} - 90\ \text{J} - 98.1\ \text{J} = 505\ \text{J} \qquad \textit{Ans.}$$

# Kinetic Energy of a Particle, Principle of Work and Energy

Kinetic energy is defined in expression of

$$\boldsymbol{T = \frac{1}{2}\, mv^2}$$

Unit

$$Kg.\left(\frac{m}{s}\right)^2 = \frac{Kg\ .m}{s^2}\ .\ m = N\ .m = J$$

OR = lb . ft

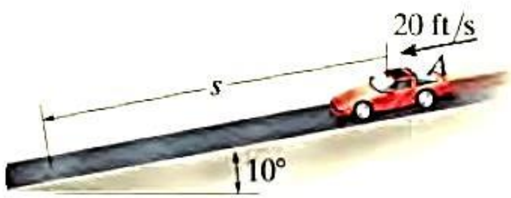
Then the work and energy of a particle moves from point 1 to point 2 can be express by the equation:

$$\mathbf{U = T_2 - T_1}$$

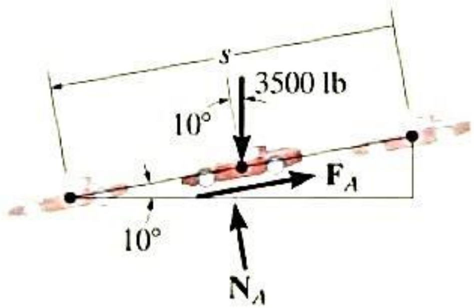
The work and energy also can be written as

$$\mathbf{T_1 + \sum U = T_2}$$

# EXAMPLE




The 3500-lb automobile travels down the 10° inclined road at a speed of 20 ft/s. If the driver jams on the brakes, causing his wheels to lock, determine how far $s$ the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.




SOLUTION

**Work**

$+\nwarrow \Sigma F_n = 0; \quad N_A - 3500 \cos 10° \text{ lb} = 0 \qquad N_A = 3446.8 \text{ lb}$

Thus,

$$F_A = \mu_k N_A = 0.5\,(3446.8 \text{ lb}) = 1723.4 \text{ lb}$$

**Principle of Work and Energy.**

$$T_1 + \Sigma U_{1\text{-}2} = T_2$$

$$\frac{1}{2}\left(\frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(20 \text{ ft/s})^2 + 3500 \text{ lb}(s \sin 10°) - (1723.4 \text{ lb})s = 0$$

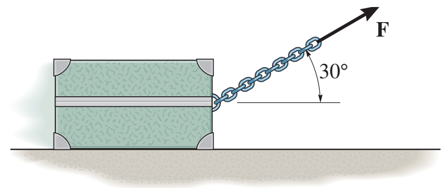
Solving for $s$ yields

$$s = 19.5 \text{ ft} \qquad \textit{Ans.}$$

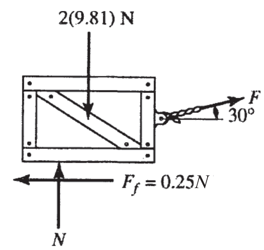
H. W. : Chapter 14: 1, 6, 7, 8, 10

**14–1.**

The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.




## SOLUTION




***Equation of Motion:*** Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25N$. Applying Eq. 13–7, we have

$$+\uparrow \sum F_y = ma_y; \qquad N + 100 \sin 30^\circ - 20(9.81) = 20(0)$$

$$N = 146.2 \text{ N}$$

***Principle of Work and Energy:*** The horizontal component of force $F$ which acts in the direction of displacement does *positive* work, whereas the friction force $F_f = 0.25(146.2) = 36.55$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction $N$, the vertical component of force $F$ and the weight of the crate do not displace hence do no work. Applying Eq.14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

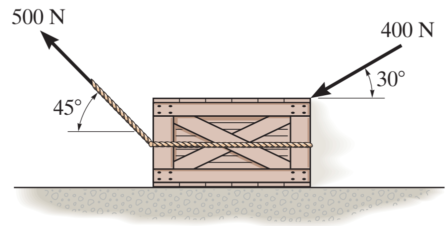
$$\frac{1}{2}(20)(8^2) + \int_{15\text{ m}}^{25\text{ m}} 100 \cos 30^\circ\, ds - \int_{15\text{ m}}^{25\text{ m}} 36.55\, ds = \frac{1}{2}(20)v^2$$

$v = 10.7$ m/s **Ans.**

**Ans:**
$v = 10.7$ m/s

***14–4.**

The 100-kg crate is subjected to the forces shown. If it is originally at rest, determine the distance it slides in order to attain a speed of $v = 8\text{ m/s}$. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.




## SOLUTION

***Work.*** Consider the force equilibrium along the $y$ axis by referring to the FBD of the crate, Fig. $a$,

$$+\uparrow \Sigma F_y = 0; \qquad N + 500 \sin 45^\circ - 100(9.81) - 400 \sin 30^\circ = 0$$

$$N = 827.45\text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.2(827.45) = 165.49$ N. Here, $F_1$ and $F_2$ do positive work whereas $F_f$ does negative work. $W$ and $N$ do no work

$$U_{F_1} = 400 \cos 30^\circ \text{ s} = 346.41 \text{ s}$$

$$U_{F_2} = 500 \cos 45^\circ \text{ s} = 353.55 \text{ s}$$

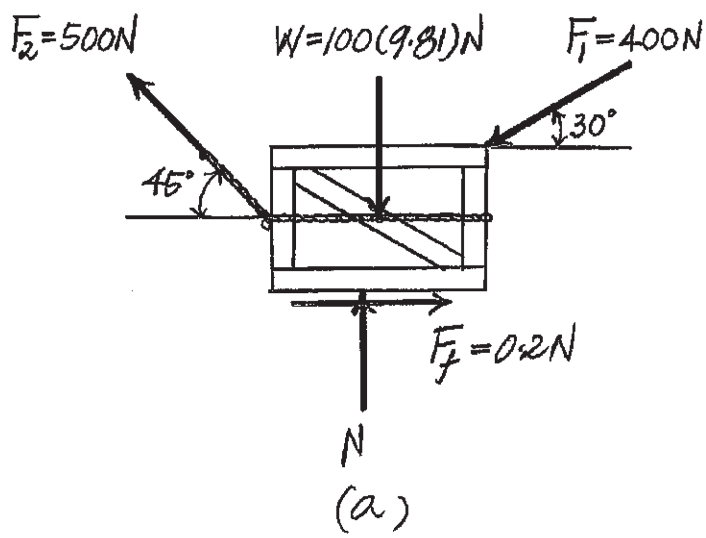
$$U_{F_f} = -165.49 \text{ s}$$

***Principle of Work And Energy.*** Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 346.41 \text{ s} + 353.55 \text{ s} + (-165.49 \text{ s}) = \frac{1}{2}(100)\left(8^2\right)$$

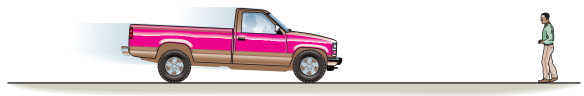
$$s = 5.987\text{ m} = 5.99\text{ m} \qquad \textbf{Ans.}$$



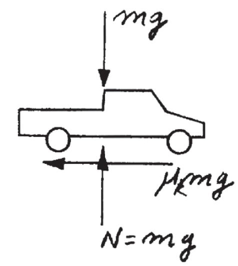

**Ans:**
$s = 5.99$ m

**14–6.**

When the driver applies the brakes of a light truck traveling 40 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?



## SOLUTION




$$40 \text{ km/h} = \frac{40(10^3)}{3600} = 11.11 \text{ m/s} \qquad 80 \text{ km/h} = 22.22 \text{ m/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m(11.11)^2 - \mu_k mg(3) = 0$$

$$\mu_k g = 20.576$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m(22.22)^2 - (20.576)m(d) = 0$$

$$d = 12 \text{ m}$$ **Ans.**

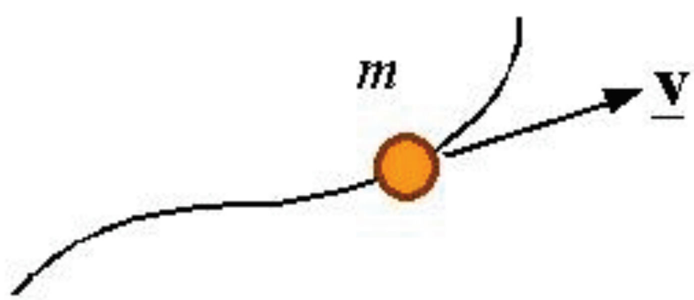
**Ans:**
$d = 12$ m

# Principle of Linear Impulse and Momentum

**Momentum**: is a vector quantity that is the product of the mass and the velocity of an object or particle. (Momentum: is mass in motion)
The unit of momentum is (kg.m/s).

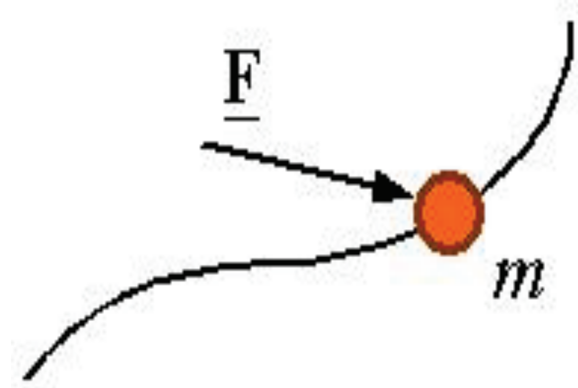
$$p = m * v$$

p = momentum
m = mass
v = velocity



**Impulse:** is a term that quantifies the overall effect of a force acting over time. It is conventionally given the symbol **I** and expressed in Newton-seconds. For a constant force,

$$I = F.\ \Delta t$$




Impulse of a force from time t1 to t2: The integral of the force over the time interval of concern is its impulse. The impulse of a force is a vector given by the integral

$$\mathbf{I} \equiv \int_{t_1}^{t_2} \mathbf{F}\,dt$$

## The Momentum-Impulse Theorem:

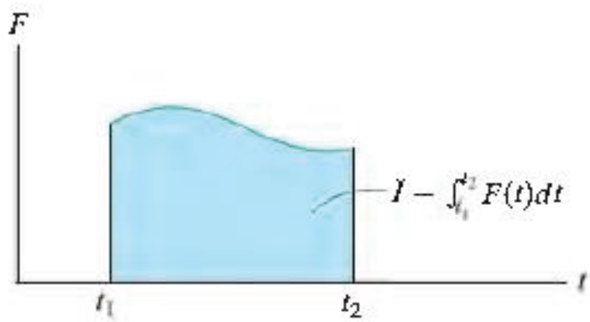
states that in order to change the momentum of an object, one must exert an impulse

(change in momentum) = (impulse)

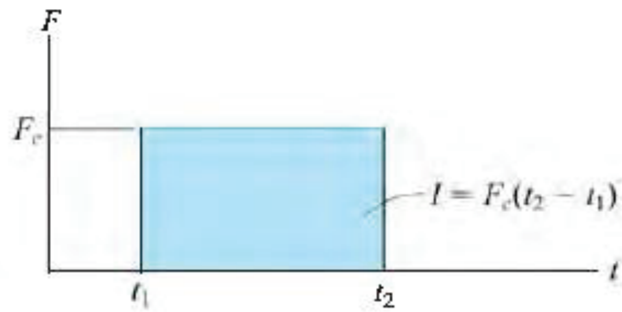
$p_{final} - p_{initial}$ = (force) * (time)

$m*v_{final} - m*v_{initial}$ = (force) * (time)

$$\mathbf{m}\,\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}\,dt = \mathbf{m}\,\mathbf{v}_2$$




Variable Force

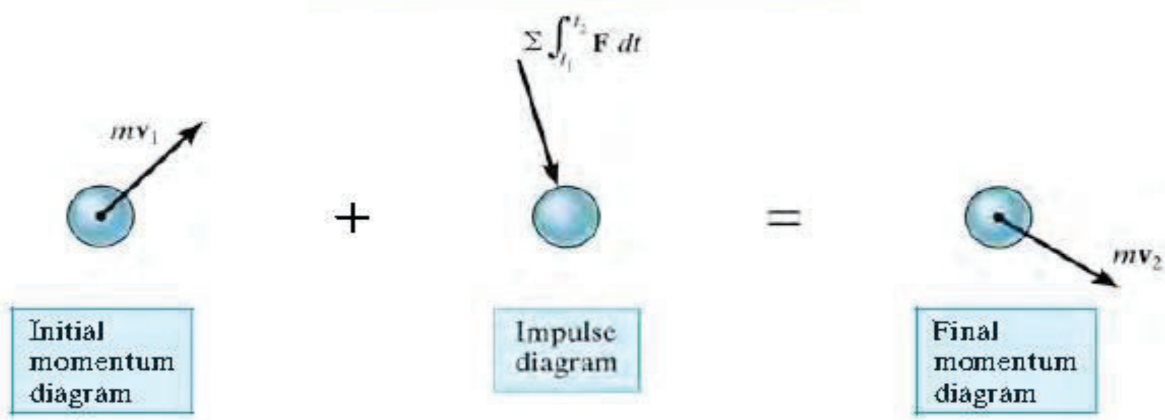



Constant Force

If each of the vectors is resolved into its x, y components, we can write the following two scalar equations of linear impulse and momentum.

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$



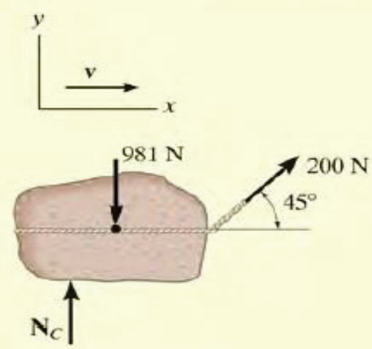
## EXAMPLE 15.1

The 100-kg stone shown in Fig. 15–4*a* is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied to the stone for 10 s, determine the final velocity and the normal force which the surface exerts on the stone during this time interval.

(a)

### SOLUTION

**Principle of Impulse and Momentum.**



(b)

$(\overset{+}{\rightarrow})$

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$0 + 200\text{ N} \cos 45°(10\text{ s}) = (100\text{ kg})v_2$$

$$v_2 = 14.1\text{ m/s} \qquad \textit{Ans.}$$

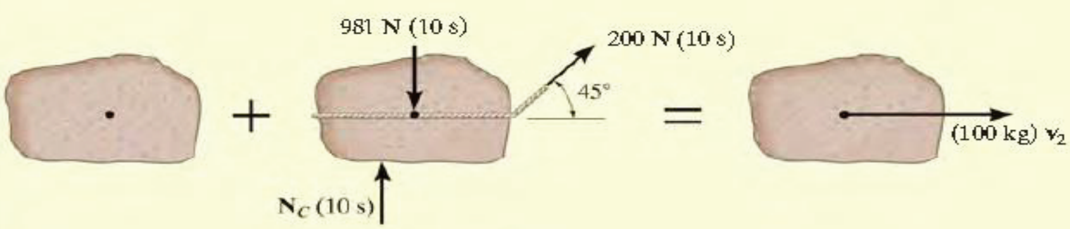
$(+\uparrow)$

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

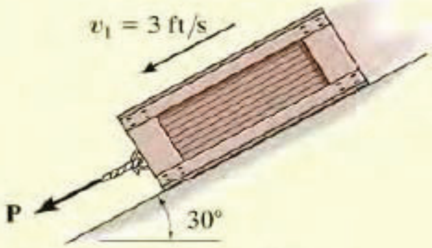
$$0 + N_C(10\text{ s}) - 981\text{ N}(10\text{ s}) + 200\text{ N} \sin 45°(10\text{ s}) = 0$$

$$N_C = 840\text{ N} \qquad \textit{Ans.}$$

**NOTE:** Since no motion occurs in the $y$ direction, direct application of the equilibrium equation $\Sigma F_y = 0$ gives the same result for $N_C$.



## EXAMPLE 15.2




(a)

The 50-lb crate shown in Fig. 15–5*a* is acted upon by a force having a variable magnitude $P = (20t)$ lb, where $t$ is in seconds. Determine the crate's velocity 2 s after **P** has been applied. The initial velocity is $v_1 = 3$ ft/s down the plane, and the coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.

### SOLUTION

**Principle of Impulse and Momentum.** Applying Eqs. 15–4 in the $x$ direction, we have

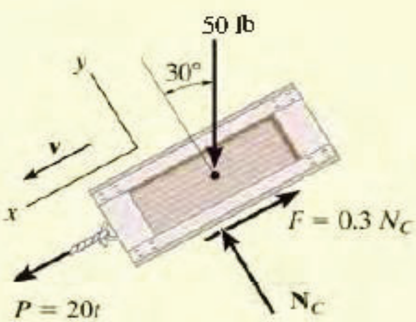
$$(+\swarrow) \qquad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$\frac{50 \text{ lb}}{32.2 \text{ ft/s}^2}(3 \text{ ft/s}) + \int_0^{2\text{ s}} 20t\, dt - 0.3N_C(2 \text{ s}) + (50 \text{ lb}) \sin 30°(2 \text{ s}) = \frac{50 \text{ lb}}{32.2 \text{ ft/s}^2} v_2$$

$$4.658 + 40 - 0.6N_C + 50 = 1.553v_2$$

The equation of equilibrium can be applied in the $y$ direction.

$$+\nwarrow \Sigma F_y = 0; \qquad N_C - 50 \cos 30° \text{ lb} = 0$$



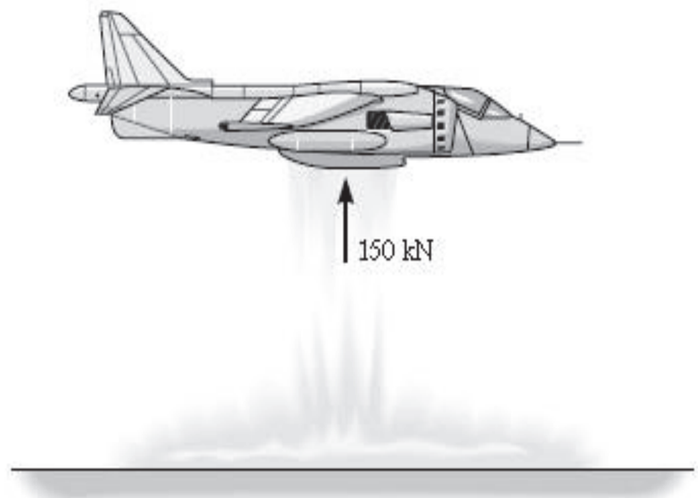

(b)

Solving,

$$N_C = 43.30 \text{ lb}$$

$$v_2 = 44.2 \text{ ft/s} \swarrow \qquad \textit{Ans.}$$

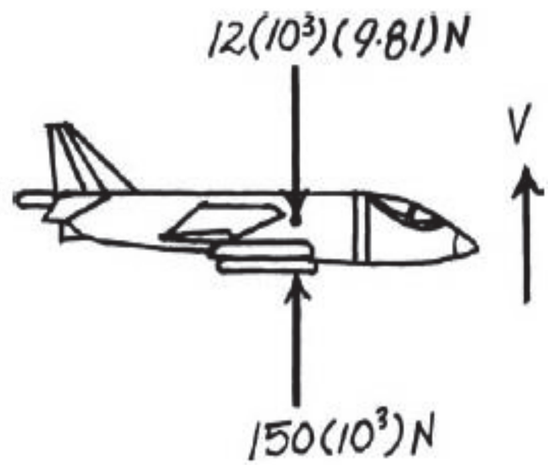
**15–2.** The 12-Mg "jump jet" is capable of taking off vertically from the deck of a ship. If its jets exert a constant vertical force of 150 kN on the plane, determine its velocity and how high it goes in $t = 6$ s, starting from rest. Neglect the loss of fuel during the lift.




$(+\uparrow)$ $\quad m(v_y)_1 + \Sigma \int F_y\, dt = m(v_y)_2$

$$0 + 150(10^3)(6) - 12(10^3)(9.81)(6) = 12(10^3)v$$

$$v = 16.14 \text{ m/s} = 16.1 \text{ m/s}$$ **Ans.**




$(+\uparrow)$ $\quad v = v_0 + a_c t$

$$16.14 = 0 + a(6)$$

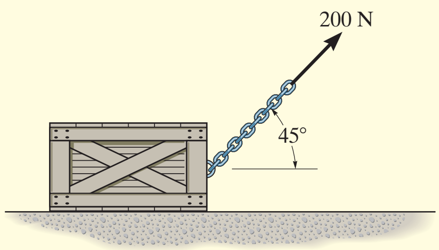
$$a = 2.690 \text{ m/s}^2$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$s = 0 + 0 + \frac{1}{2}(2.690)(6)^2$$

$$s = 48.4 \text{ m}$$ **Ans.**

## EXAMPLE 15.1



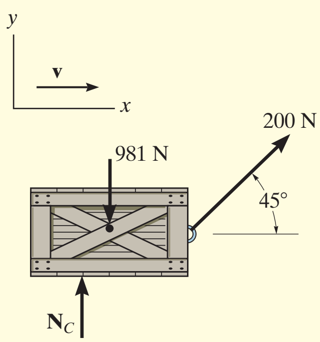

(a)

The 100-kg crate shown in Fig. 15–5*a* is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied for 10 s, determine the final velocity and the normal force which the surface exerts on the crate during this time interval.

### SOLUTION

This problem can be solved using the principle of impulse and momentum since it involves force, velocity, and time.

**Free-Body Diagram.** See Fig. 15–5*b*. Since all the forces acting are *constant*, the impulses are simply the product of the force magnitude and 10 s [$\mathbf{I} = \mathbf{F}_c(t_2 - t_1)$]. Note the alternative procedure of drawing the crate's impulse and momentum diagrams, Fig. 15–5*c*.




(b)

**Principle of Impulse and Momentum.** Applying Eqs. 15–4 yields

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + 200\text{ N}\cos 45°(10\text{ s}) = (100\text{ kg})v_2$$

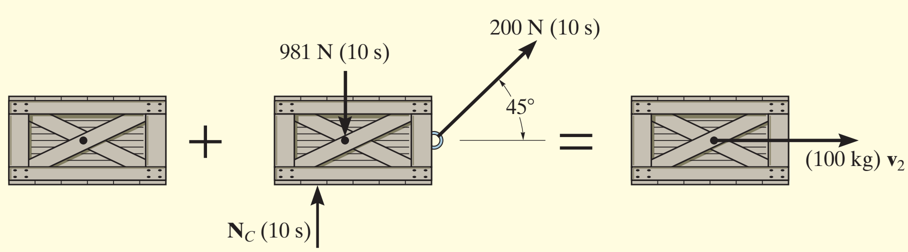
$$v_2 = 14.1\text{ m/s} \qquad \textit{Ans.}$$

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$0 + N_C(10\text{ s}) - 981\text{ N}(10\text{ s}) + 200\text{ N}\sin 45°(10\text{ s}) = 0$$

$$N_C = 840\text{ N} \qquad \textit{Ans.}$$

**NOTE:** Since no motion occurs in the $y$ direction, direct application of the equilibrium equation $\Sigma F_y = 0$ gives the same result for $N_C$. Try to solve the problem by first applying $\Sigma F_x = ma_x$, then $v = v_0 + a_c t$.

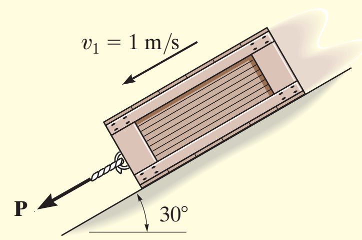



(c)

**Fig.15–5**

## EXAMPLE 15.2

The 25-kg crate shown in Fig. 15–6*a* is acted upon by a force having a variable magnitude $P = (100t)$ N, where $t$ is in seconds. Determine the crate's velocity 2 s after **P** has been applied. The initial velocity is $v_1 = 1$ m/s down the plane, and the coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.



(a)

### SOLUTION

**Free-Body Diagram.** See Fig. 15–6*b*. Since the magnitude of force $P = 100t$ varies with time, the impulse it creates must be determined by integrating over the 2-s time interval.

**Principle of Impulse and Momentum.** Applying Eqs. 15–4 in the $x$ direction, we have

$$(+\swarrow) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$(25\text{ kg})(1\text{ m/s}) + \int_0^{2\text{ s}} 100t\,dt - 0.3N_C(2\text{ s}) + (25\text{ kg})(9.81\text{ m/s}^2)\sin 30°(2\text{ s}) = (25\text{ kg})\,v_2$$

$$25 + 200 - 0.6N_C + 245.25 = 25v_2$$

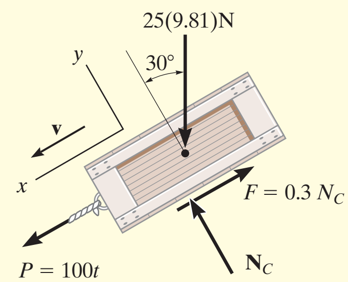
The equation of equilibrium can be applied in the $y$ direction. Why?

$$+\nwarrow\Sigma F_y = 0; \qquad N_C - 25(9.81)\cos 30°\text{ N} = 0$$

Solving,

$$N_C = 212.39\text{ N}$$

$$v_2 = 13.7\text{ m/s}\swarrow \qquad \textit{Ans.}$$



(b)

**Fig. 15–6**

**NOTE:** We can also solve this problem using the equation of motion. From Fig. 15–6*b*,

$$+\swarrow\Sigma F_x = ma_x; \quad 100t - 0.3(212.39) + 25(9.81)\sin 30° = 25a$$

$$a = 4t + 2.356$$

Using kinematics

$$+\swarrow dv = a\,dt; \qquad \int_{1\text{ m/s}}^{v} dv = \int_0^{2\text{ s}} (4t + 2.356)dt$$

$$v = 13.7\text{ m/s} \qquad \textit{Ans.}$$

By comparison, application of the principle of impulse and momentum eliminates the need for using kinematics ($a = dv/dt$) and thereby yields an easier method for solution.