وزارة التعليم العالي والبحث العلمي

جامعة الفرات الأوسط التقنية

المعهد التقني كربلاء

قسم التقنيات الميكانيكية/ الانتاج

الحقيبة التعليمية لمادة

تقنية أجزاء المكائن

المرحلة الثانية

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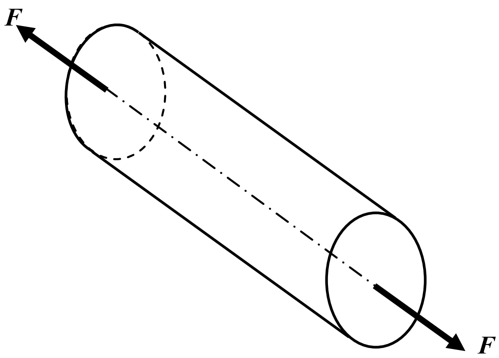
Stresses:

The word stress is used to indicate **force** per unit **area**.

$$\sigma = \frac{F}{A}$$



σ : **Stress (N/m^2)**

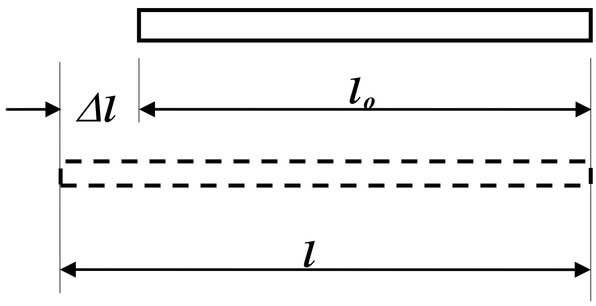
F : **Force (N)**

A : **Area (m^2)**

Strain:

A strain is measured of the deformation of the body.

$$\varepsilon = \frac{l - l_o}{l_o} = \frac{\Delta l}{l_o}$$



ε : **strain**

Stress – Strain Diagram:

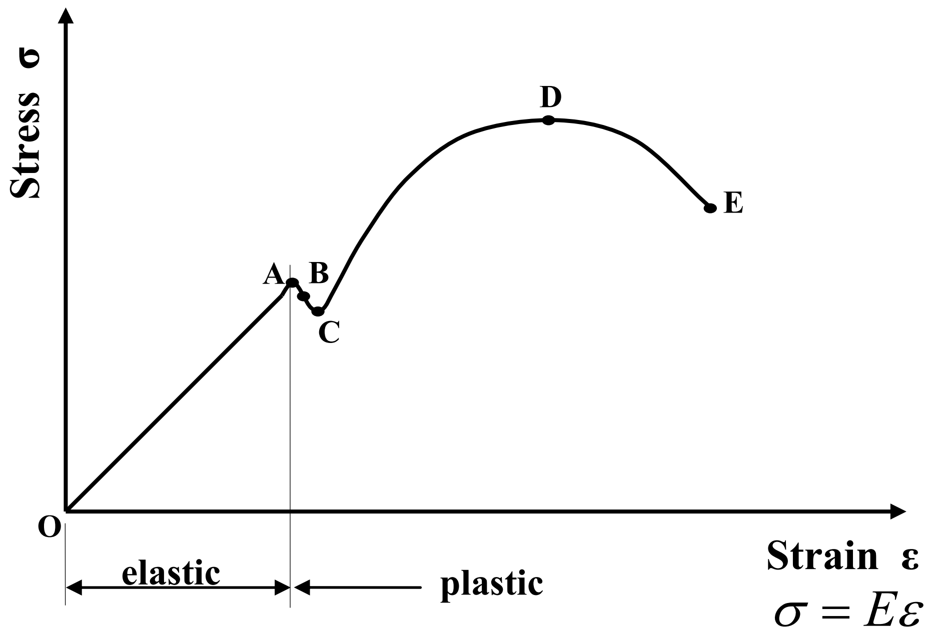
This curve is done by a tensile test to a specimen with a determined dimension, with increasing tensile force gradually and measuring the length of the specimen until fracture.

OA : proportional limit

$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

E : young modulus
Point B : Elastic limit
Point C : yield point
At this point the deformation stay after removing applied force



Point D: ultimate tensile stress
Point E: fracture
Beyond point D the external force decrease because of decreasing in cross section area (neck)

$$\sigma = E\varepsilon \Rightarrow E = \frac{\sigma}{\varepsilon} = \frac{\frac{F}{A}}{\frac{\Delta\ell}{\ell_\circ}}$$

$$\boxed{\Delta\ell = \frac{F * \ell_\circ}{E * A} = \delta}$$

Safety Factor: S.F

It is a relation between critical stress (yield, ultimate) and allowable stress (calculated)

$$S.F = \frac{\sigma_{critical}}{\sigma_{allowable}}$$

For ductile material (steel, bronze, brass ……) we use yield stress

$$S.F = \frac{\sigma_y}{\sigma_{all}} = \frac{\sigma_y}{\sigma_{cal}}$$

For brittle material (cast iron) we use ultimate stress

$$S.F = \frac{\sigma_{ult}}{\sigma_{all}} = \frac{\sigma_{ult}}{\sigma_{cal}}$$

Factor of safety depend upon the following

1- Degree of economy.
2- Value of strength, yield, and ultimate endurance limits.
3- Load condition (static, dynamic, shock).
4- Degree of accuracy.
5- Importance of machine part.
6- Degree of safety to human life.

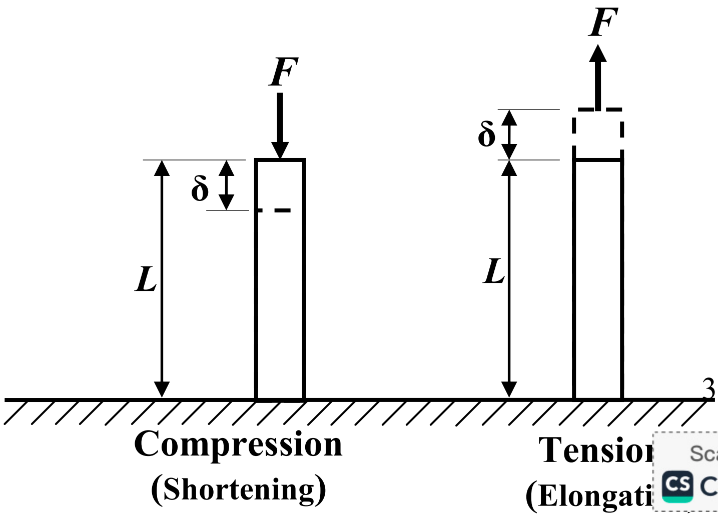
Types of stresses:

A) Tensile and Compressive stress: σ_t, σ_c

Under the effect of this stress, the member will have total elogation or shortening

$$\sigma = \frac{F}{A} \quad \text{N/m}^2$$

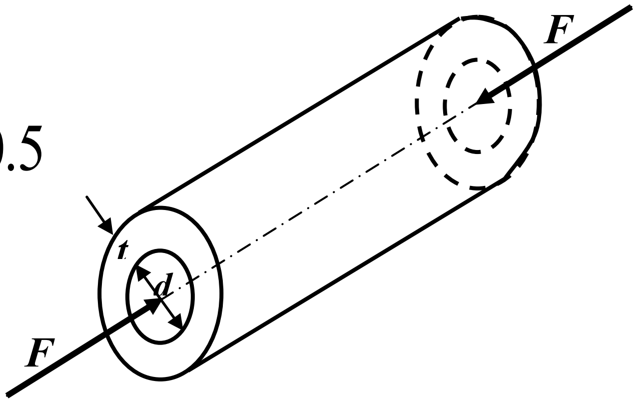
F $\perp$ A



Example (1):

Hollow cylinder with length (L =50 cm), thickness (t =0.25 cm) effected by compressive force (F = 2.5 KN) if the compression stress of material (σ_c =70 MN/m^2), modulus of elasticity (E = 200 GN/m^2). Determine the shortening in the length of cylinder.

Solution :



$$D = d + 2 * t = d + 0.5 \quad \text{cm}$$

$$A = \frac{\pi}{4}\left[D^2 - d^2\right]$$

$$A = \frac{\pi}{4}\left[(d + 0.5)^2 - d^2\right]$$

$$A = \frac{\pi}{4}\left[d^2 + d + 0.25 - d^2\right] = \frac{\pi}{4}(d + 0.25) \quad \text{cm}^2$$

$$A = \frac{\pi}{4}(d + 0.25) * 10^{-4} \quad \text{m}^2$$

$$F = 2.5 * 10^3 \quad \textbf{N}$$

$$\sigma_C = 70 * 10^6 \quad \textbf{N/m}^2$$

$$E = 200 * 10^9 \quad \textbf{N/m}^2$$

$$\sigma_c = \frac{F}{A} \qquad 70 * 10^6 = \frac{2.5 * 10^3}{\frac{\pi}{4}(d + 0.25) * 10^{-4}}$$

$$d = 0.2047 \quad \textbf{m}$$

$$A = \frac{\pi}{4}(0.2047 + 0.25) * 10^{-4} = 0.3571 * 10^{-4} \quad \textbf{m}^2$$

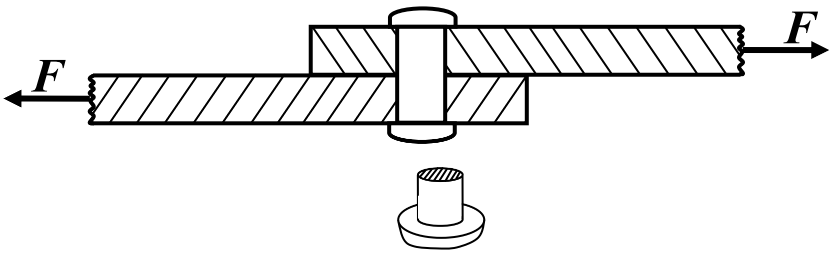
$$\delta = \frac{F * \ell}{E * A} = \frac{2.5 * 10^3 * 0.5}{200 * 10^9 * 0.3571 * 10^{-4}}$$

$\delta = 0.0002$ **m**

$\delta = 0.2$ **mm**

B) Shear stress: τ

If we have two forces effect on the body equal in magnitude and opposite in direction causing relative sliding or slipping of adjacent positions of the body.



$\tau = \frac{F}{A}$ **(N/m²)**

F ∥ A

τ : **Shear Stress (N/m²)**

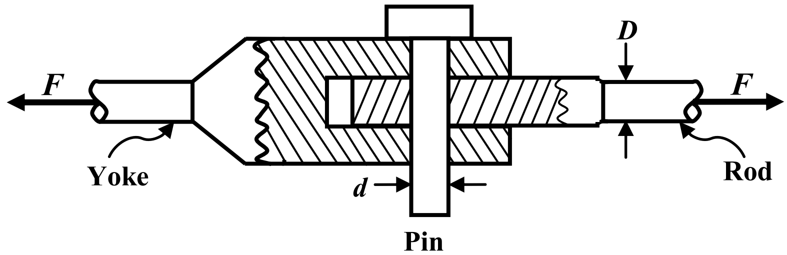
Example (2):

Determine the diameter of the rod (D) and the diameter of the pin (d) for the knuckle joint. If the transmitted force from one side to another through the pin is (80 KN) tension stress for the rod material is (σ_t = 100 MN/m²) and the shear stress for the pin material is (τ = 80 MN/m²).

Solution:

For the Rod

$$\sigma_t = \frac{F}{A}$$



$\sigma_t = 100 * 10^6$ **(N/m²)**

$F = 80 * 10^3$ N

$$A = \frac{\pi}{4} D^2$$

$$100 * 10^6 = \frac{80 * 10^3}{\frac{\pi}{4} D^2} \quad \Longrightarrow \quad D = 0.0319 \text{ m}$$

$$D = 31.9 \text{ mm}$$

For the Pin

$$\tau = \frac{F}{2A}$$

$\tau = 80 * 10^6$ **(N/m²)**

$$A = \frac{\pi}{4} d^2$$

$$80 * 10^6 = \frac{80 * 10^3}{2 * \frac{\pi}{4} d^2} \quad \Longrightarrow \quad d = 0.0252 \text{ m}$$

$$d = 25.2 \text{ mm}$$

C) Bearing stress: $\sigma_{bearing}$

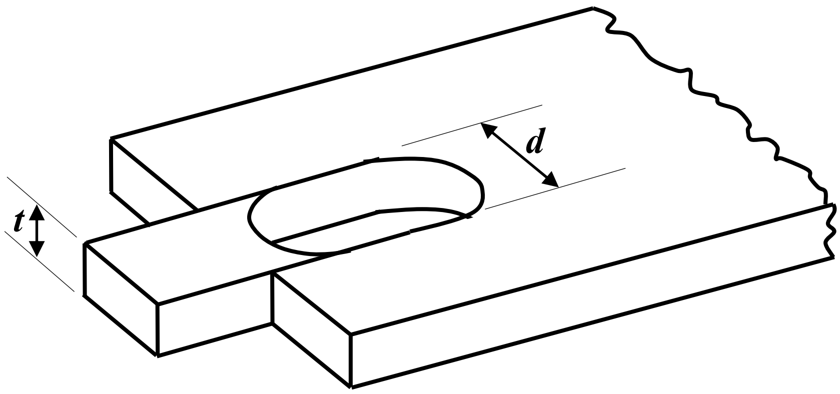
When one object presses against another is referred as bearing stress or (crushing stress)

$$\sigma_{bearing} = \frac{F}{A_{proj}} \qquad (N/m^2)$$

F ⊥ Aproj

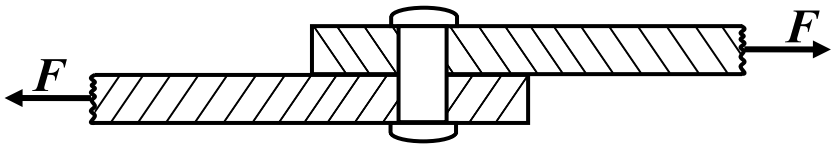
$\sigma_{Bearing}$: **Bearing Stress (N/m²)**

$$A_{Proj} = t * d$$



Example (3):

Two plates is riveted by lap joint as shown the thickness of plate is ***(16 mm)*** and the diameter of rivet is ***(2.5 cm)*** find the crushing stress when ***(4800 N)*** is applied , and find the shear stress for rivet.



Solution:

$$t = 16mm = 0.016m$$

$$d = 2.5cm = 0.025m$$

Bearing stress

$$\sigma_{bearing} = \frac{F}{A_{proj}} = \frac{F}{t*d} = \frac{4800}{0.016*0.025} = 12000000\ \frac{N}{m^2}$$

$$\sigma_{bearing} = 12\frac{MN}{m^2}$$

<u>Shear stress</u>

$$\tau = \frac{F}{A} = \frac{4800}{\frac{\pi}{4}(0.025)^2} = 9778480\ \frac{N}{m^2}$$

$$\tau = 9.78\frac{MN}{m^2}$$

Joint:

Some machine parts due to the purpose of holding, adjustment inspection, repair and replacement must be constructed to be read for conection or disconection.

Types of joint:

1 permanent joint

A - Rivet joint
B - Welding joint
C - Pressing joint

2 non permanent joint

A - Screw and bolt joint
B - Keys
C - Shaft coupling
D - Cutter pin joint

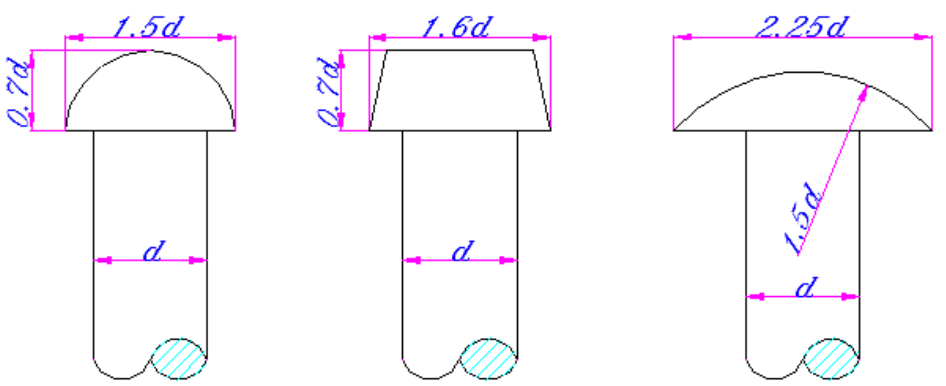
Rivet joints:

It is a permanent fastenings used for various engineering structures such as boilers, bridges, cranes, ships, cars.

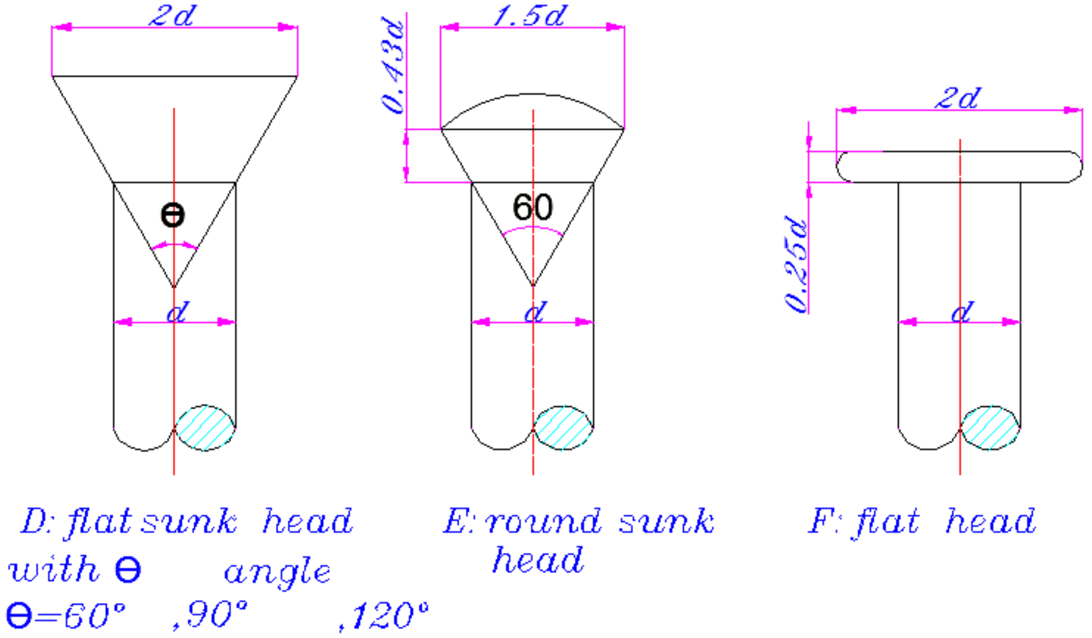
Material: *the rivet are made from*

A- mild steel used for ships, car bodies
B- Aluminium and copper used for airplane bodies

Types of rivet heads and standard dimensions:

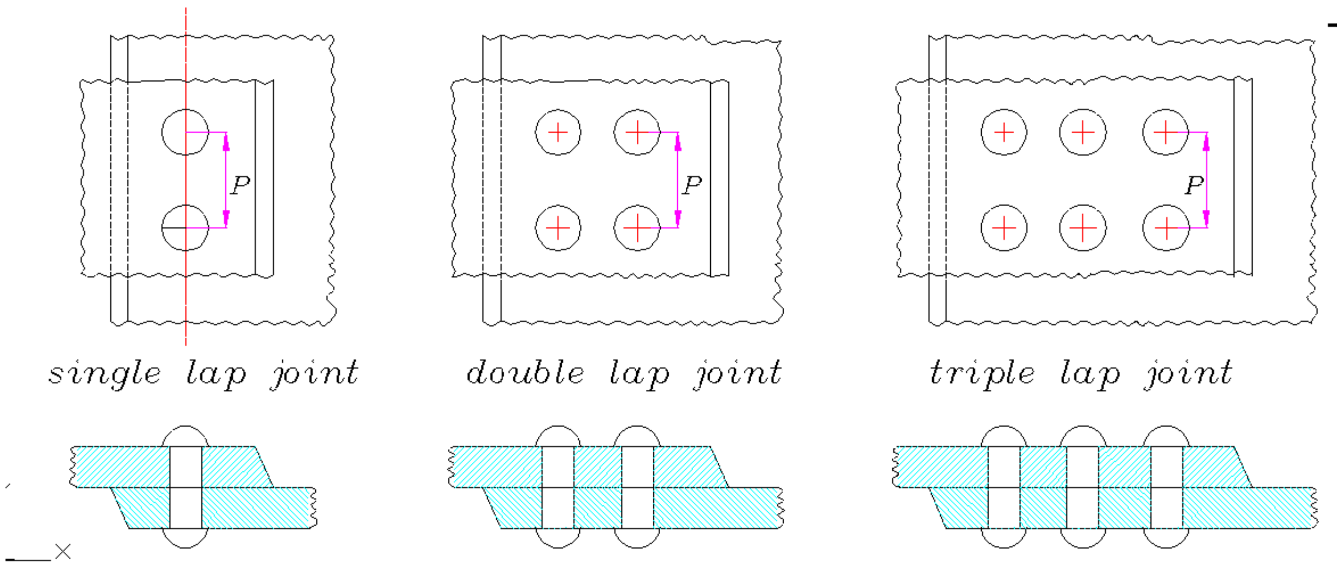


A: snap head B: pan head C: curve head or mushroom head



Types of riveted joints: *there are two types*

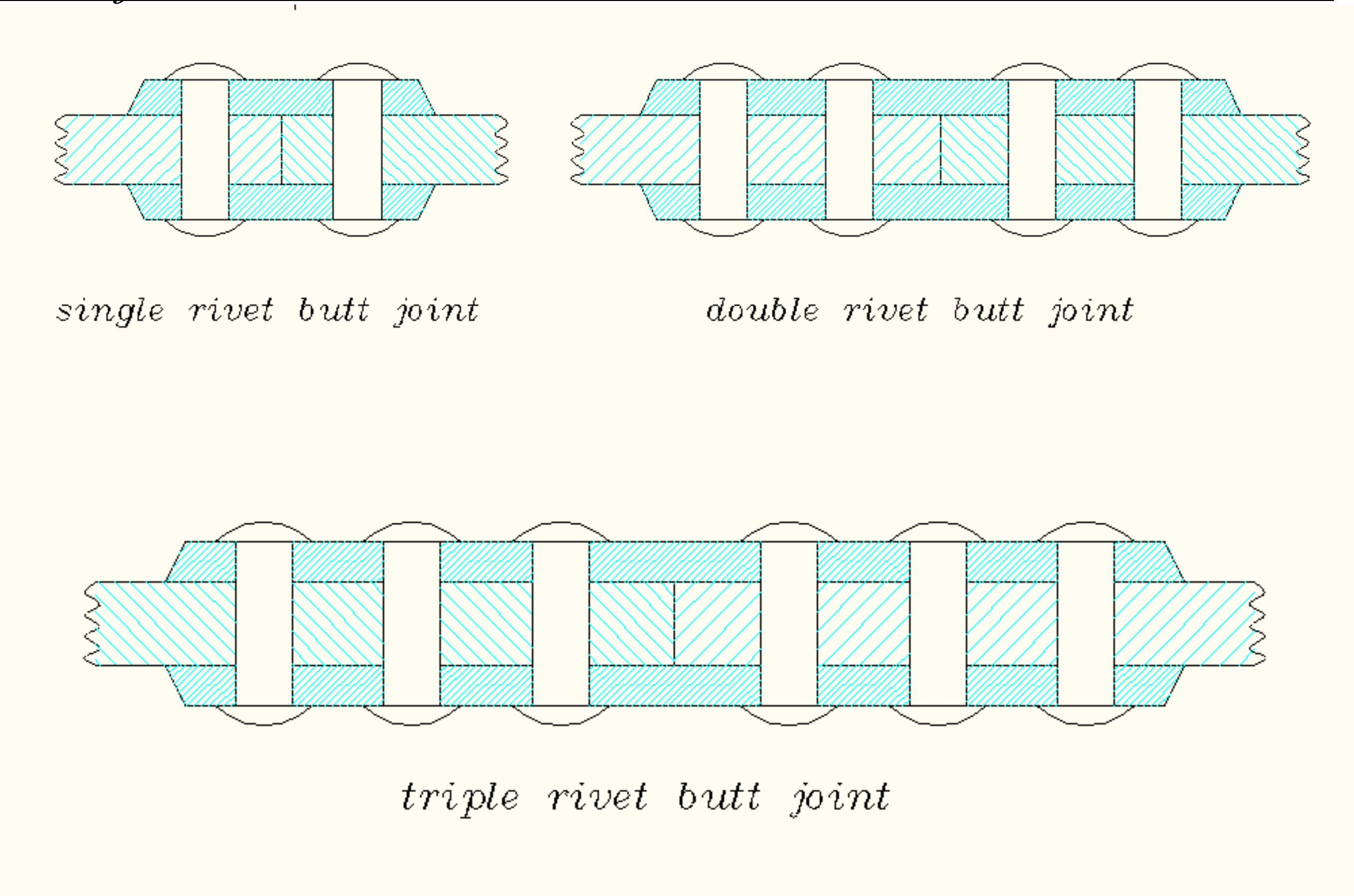
1- Lap joints



2- ***Butt joints:*** *in butt riveting the plates are kept in alignment and a butt strap or cover plate is placed over the joint and riveted to each plate.*

Types of Butt joints:

1. *single rivet butt joint*
2. *double rivet butt joint*
3. *triple rivet butt joint*



Design of rivet joints:

The rivet joint is subjected to a tension force and may be design on the base of following stresses

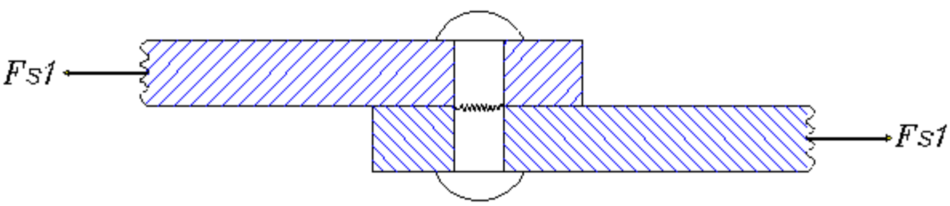
1- ***Shear stress for rivet:***

A- Lap joint:

$$\tau = \frac{F_{S1}}{n \cdot A}$$

$$F_{s1} = n \cdot A \cdot \tau$$

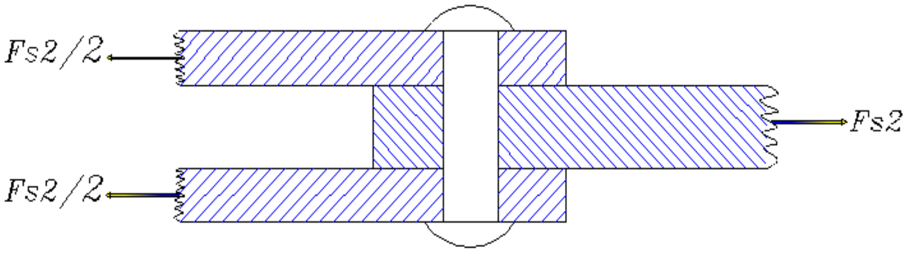
$$\boxed{F_{s1} = n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau}$$



B- Butt joint:

$$\tau = \frac{F_{S2}}{2 \cdot n \cdot A}$$

$$F_{s2} = 2 \cdot n \cdot A \cdot \tau$$



$$F_{s2} = 2 \cdot n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau$$

Where:
F_{s1} ,F_{s2} : shear force *(N)*
d: diameter of rivet
τ : shear stress ***(N/m²)***
n : number of rows

Note:

1- In practice we take $F_{S2} = 1.875 * F_{S1}$

2- in case of boiler joints $F_{S2} = 1.75 * F_{S1}$

3- in case of n- row of rivets

$$F_{s1} = n \cdot A \cdot \tau$$

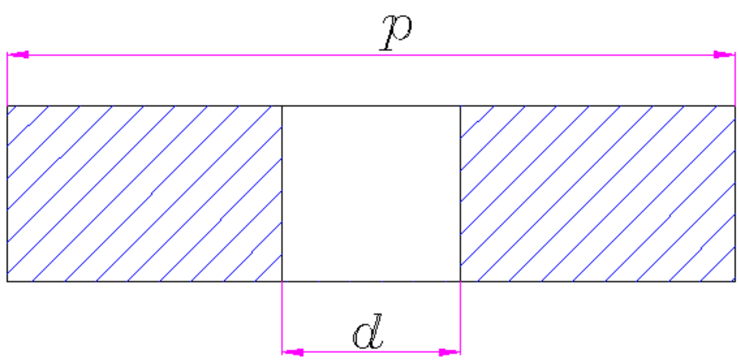
$$F_{s2} = 2 \cdot n \cdot A \cdot \tau$$

Where: *n* : number of rows

2- Tension stress for plate:

$$\sigma_t = \frac{F_t}{A_t}$$

$$F_t = \sigma_t \cdot A_t$$

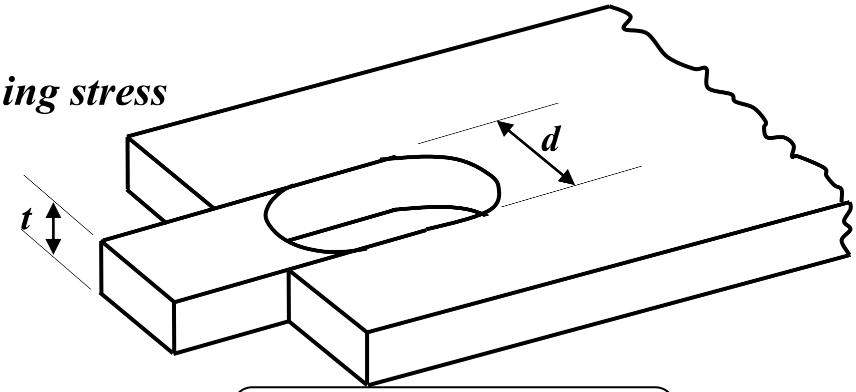


$$F_t = \sigma_t \cdot (P - d) \cdot t$$

Where:
F_t: tensile force *(N)*
p: pitch
σ_t: tensile stress ***(N/m²)***

Crushing stress of plate:

The plate is subjected to crushing stress due to tensile force



$$\sigma_{cr} = \frac{F_{cr}}{n \cdot A_{cr}}$$

$$F_{cr} = \sigma_{cr} \cdot n \cdot A_{cr} \quad \Rightarrow \quad F_{cr} = \sigma_{cr} \cdot n \cdot t \cdot d$$

Where:
F_{cr}: crushing force *(N)*
σ_{cr}: crushing stress ***(N/m²)***

Efficiency of rivet joints:

It is the ability to bear shear, tension and crushing forces. It may be estimated by calculation the cross section area between rivets hole centres as follow

$$A = p \cdot t$$

$$F = \sigma_t \cdot A$$

$$F = \sigma_t \cdot p \cdot t$$

1 Shearing efficiency

A Lap joint

$$\eta_s = \frac{F_{s1}}{F} \times 100\%$$

$$\eta_s = \frac{n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau}{p \cdot t \cdot \sigma_t} \times 100\%$$

(B) Butt joint

$$\eta_s = \frac{F_{s2}}{F} \times 100\%$$

$$\boxed{\eta_s = \frac{2 \cdot n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau}{p \cdot t \cdot \sigma_t} \times 100\%}$$

(2) Tension efficiency

$$\eta_t = \frac{F_t}{F} \times 100\% = \frac{(p-d) \cdot t \cdot \sigma_t}{p \cdot t \cdot \sigma_t} \times 100\%$$

$$\boxed{\eta_t = \frac{(p-d)}{p} \times 100\%}$$

(3) crushing efficiency

$$\eta_{cr} = \frac{F_{cr}}{F} \times 100\% = \frac{n \cdot t \cdot d \cdot \sigma_{cr}}{p \cdot t \cdot \sigma_t} \times 100\%$$

$$\boxed{\eta_{cr} = \frac{n \cdot d \cdot \sigma_{cr}}{p \cdot \sigma_t} \times 100\%}$$

Practical equation for designing the rivet joints:

1- Diameter of rivet depend on the thickness of plate

A: when $t \geq 8mm$

$$\boxed{d = 6\sqrt{t}} \quad mm$$

B: when $t < 8mm$

Lap joint

$$F_{s1} = F_{cr} \Longrightarrow n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = n \cdot t \cdot d \cdot \sigma_{cr} \Longrightarrow d = \frac{4 \cdot t \cdot \sigma_{cr}}{\pi \cdot \tau}$$

Butt joint

$$F_{s2} = F_{cr} \Longrightarrow 2 \cdot n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = n \cdot t \cdot d \cdot \sigma_{cr} \Longrightarrow d = \frac{2 \cdot t \cdot \sigma_{cr}}{\pi \cdot \tau}$$

2- Calculation of pitch

Lap joint

$$F_{s1} = F_t \Longrightarrow n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = (p - d) \cdot t \cdot \sigma_t \qquad p =$$

Butt joint

$$F_{s2} = F_t \Longrightarrow 2 \cdot n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = (p - d) \cdot t \cdot \sigma_t \qquad p =$$

3- The distance between the centre of rivet and the edge of plate must be more than 1.5d

4- Calculation of the thickness of cover plate in case of butt joint

A: in case of one cover plate

$$t_1 = 1.25 \cdot t$$

B: in case of two cover plate

$$t_1 = 0.6t \rightarrow 0.8t$$

Where:

t_1: thickness of cover plate

t : thickness of plate

Example (1)

*Two **(10 mm)** thick plates are to be jointed by single riveted lap joint. If the tension stress of plate materials is **(σ_t = 100 MN/m²)**, Shear stress of rivet material is **(τ = 70 MN/m²)**. Determine:*

1. *Diameter of Rivet*
2. *Rivet Pitch*
3. *Shearing Efficiency*

Solution :

(1) *because* $t \geq 8mm$

$$d = 6\sqrt{t} = 6\sqrt{10} = 18.97mm$$

(2) *the pitch take form*

$$F_{s1} = F_t \Longrightarrow n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = (p - d) \cdot t \cdot \sigma_t$$

$$1 \times \frac{\pi}{4} \times \left(\frac{18.97}{1000}\right)^2 \times 70 \times 10^6 = \left(p - \frac{18.97}{1000}\right) \times \frac{10}{1000} \times 100 \times 10^6$$

$$p = 0.03877m = 38.77mm$$

(3) *efficiency of shearing*

$$\eta_s = \frac{F_{S1}}{F} \times 100\% = \frac{n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau}{p \cdot t \cdot \sigma_t} \times 100\%$$

$$\eta_s = \frac{1 \times \frac{\pi}{4} \times \left(\frac{18.97}{1000}\right)^2 \times 70 \times 10^6}{\left(\frac{38.77}{1000}\right) \times \left(\frac{10}{1000}\right) \times 100 \times 10^6} \times 100\% = 51\%$$

Example (2)

*A double riveted lap joint is to be made between (**5 mm)** plates. If the safe working stresses are [**(σ_{cr}=110 MN/m²), (σ_t=70 MN/m²), (τ =55 MN/m²)**].calculate the rivet diameter, rivet pitch and state how the joint will fail?*

Solution :

(1) *because* $t < 8mm$

$$F_{s1} = F_{cr}$$

$$n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = n \cdot t \cdot d \cdot \sigma_{cr}$$

$$2 \times \frac{\pi}{4} \times d^2 \times 55 \times 10^6 = 2 \times 0.005 \times d \times 110 \times 10^6$$

$$d = 0.0127m = 12.7mm$$

(2) *the pitch take form*

$$F_{s1} = F_t \Longrightarrow n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = (p-d) \cdot t \cdot \sigma_t$$

$$2 \times \frac{\pi}{4} \times (0.0127)^2 \times 55 \times 10^6 = (p - 0.0127) \times 0.005 \times 70 \times 10^6$$

$$p = 0.05275m = 52.75mm$$

(3) *for fail state we must determine efficiency for all cases*

$$\eta_s = \frac{F_{S1}}{F} \times 100\% = \frac{n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau}{p \cdot t \cdot \sigma_t} \times 100\%$$

$$\eta_s = \frac{2 \times \frac{\pi}{4} \times (0.0127)^2 \times 55 \times 10^6}{(0.05275) \times (0.005) \times 70 \times 10^6} \times 100\% = 75.86\%$$

$$\eta_t = \frac{F_t}{F} \times 100\% = \frac{(p-d) \cdot t \cdot \sigma_t}{p \cdot t \cdot \sigma_t} \times 100\% = \frac{(p-d)}{p} \times 100\%$$

$$\eta_t = \frac{(0.05275 - 0.0127)}{0.05275} \times 100\% = 75.92\%$$

$$\eta_{cr} = \frac{F_{cr}}{F} \times 100\% = \frac{n \cdot t \cdot d \cdot \sigma_{cr}}{p \cdot t \cdot \sigma_t} \times 100\% = \frac{n \cdot d \cdot \sigma_{cr}}{p \cdot \sigma_t} \times 100\%$$

$$\eta_{cr} = \frac{2 \times 0.0127 \times 110 \times 10^6}{0.05275 \times 70 \times 10^6} \times 100\% = 75.67\%$$

The plate will fail in crushing

Example (3)

*Two **(15 mm)** thick plates are to be jointed by triple riveted double cover strap butt joint. If the Shear stress of rivet material is **(τ =61.7 MN/m^2)** and tension stress for plate materials is **(σ_t=82.4 MN/m^2)**. Determine:*

1. Diameter of Rivet.
2. Rivet Pitch.
*3. Cover strap thickness **(t_1)**.*
4. Shearing efficiency of Riveted.

SOLUTION :

(1) *because* $t \geq 8mm$

$$d = 6\sqrt{t} = 6\sqrt{15} = 23.2mm$$

(2) *the pitch take form*

$F_{S2} = F_t$ ⟹ $2 \cdot n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = (p - d) \cdot t \cdot \sigma_t$

$$2\times 3\times \frac{\pi}{4}\times (0.0232)^2\times 61.7\times 10^6=(p-0.0232)\times \frac{15}{1000}\times 82.4\times 10^6$$

$$p=0.1498m=149.8mm$$

(3) $t_1=0.6t\rightarrow 0.8t$

$$t_1=0.7\times t=0.7\times 15=10.5mm$$

(4) *efficiency of shearing*

$$\eta_s=\frac{F_{S2}}{F}\times 100\%=\frac{2\cdot n\cdot \frac{\pi}{4}\cdot d^2\cdot \tau}{p\cdot t\cdot \sigma_t}\times 100\%$$

$$\eta_s=\frac{2\times 3\times \frac{\pi}{4}\times (0.0232)^2\times 61.7\times 10^6}{(0.1498)\times (0.015)\times 82.4\times 10^6}\times 100\%=84.5\%$$

Welding Joint:

It is one of the permanent joint; it was obtained by heating two pieces until fused together or indirectly by using weld metal which is deposited in corner between two surfaces.

Types of welding processes:

1 ***Forge welding***

There are two types of forge welding

A - ***Manual forges welding***

In manual forge welding, the two parts are heated to plastic state and the pressure is applied by hand hammer.

B - ***Machine forges welding***

The two parts are heated to plastic state and the external pressure is applied by press machine.

2 ***Electric resistance welding :(E.R.W)***

The two parts are pressed together and current is passed from one part to other until the metal is heated to fusion temperature at the joint.

The generated heat (H) during E.R.W is

$$H = K \cdot I^2 \cdot R \cdot t$$

Where:

K*: constant*

R*: electric resistance (*Ω*)*

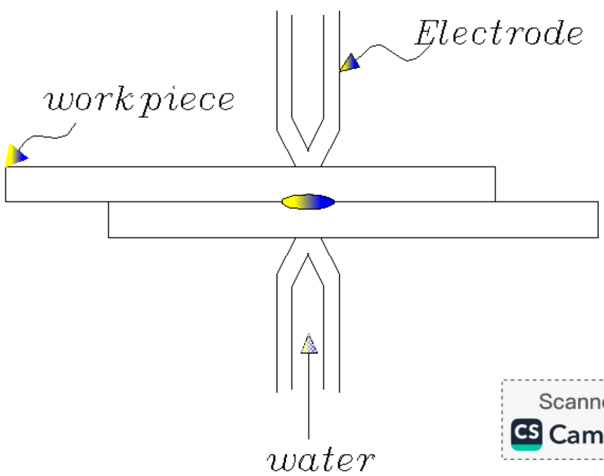
I*: electric current (A)*

t*: time (sec)*

There are three types of electric resistance welding

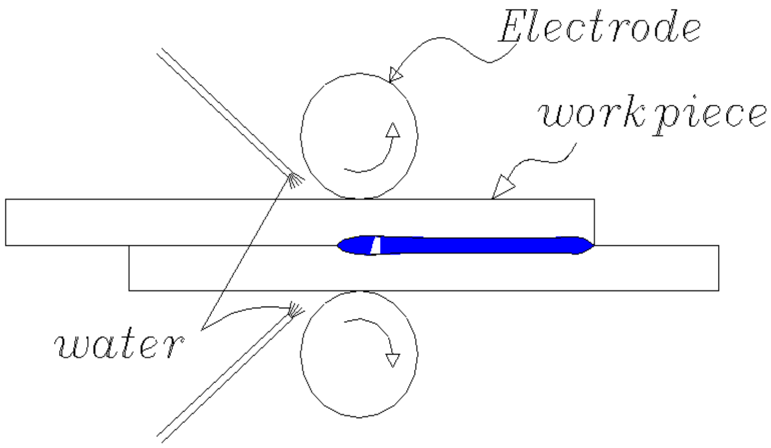
A – ***spot welding***

In which we use two-point electrodes on each sides of the cover lapped plates with Appling pressure. This type is very cheap.



B – **seam welding**

In seam welding we use rollers instead of point's electrodes. The plates are to be joint are pulled between the rollers in order to get a uniform continuous strip of welded surface.



C – **flash welding**

This type is depending on air resistance to the passing of electric current from one part to another until the metal is heated to fusion temperature.

3 Fusion (melting) welding:

It is a process of jointing two pieces in a molten state without application of mechanical pressure by heating the joint or member to temperature below the critical T-r of metal.

There types are

A – **Gas welding**

It uses oxy-hydrogen or oxy-acetylene burnt in welding torch. The edges of the work pieces are melting which on cooling results in a strong joint.

B – **Electric arc welding**

The welding temperature is developed by electric arc which is struck between work pieces and electrode which is held by operator or guided automatically.

C – **Thermit welding**

A mixture of iron oxide and aluminium called thermit is ignited and the iron oxide is reduced to molten metal. The advantage of this type is that all parts of the weld section are molten at the same time and the cooling of the weld is uniform.

Types of welding joints:

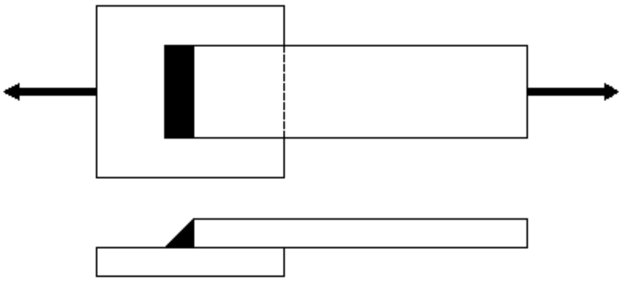
There are two types

1 Lap welds joints(fillet weld):

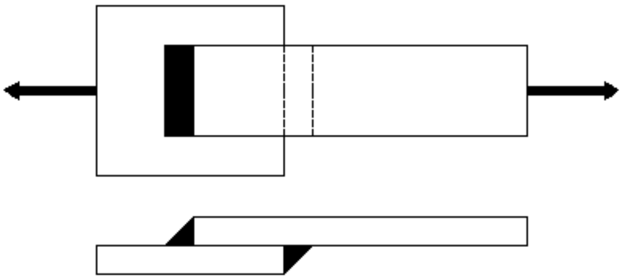
The weld metal is deposited in the corner between the two surfaces.

The types are

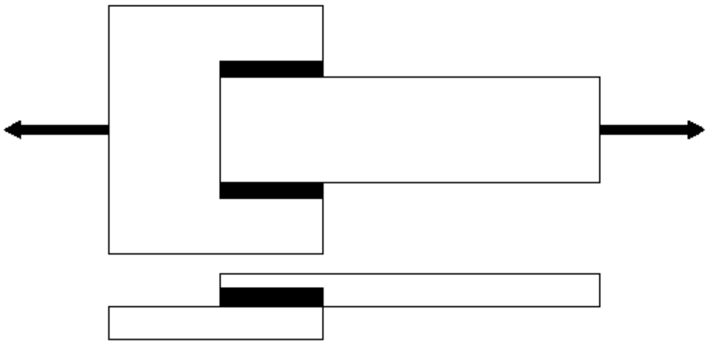
A – Single transverse fillet



B – Double transverse fillet



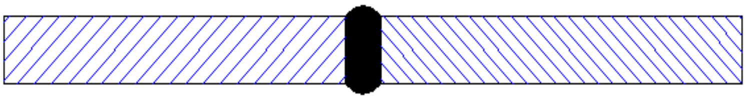
C – Parallel fillet joint



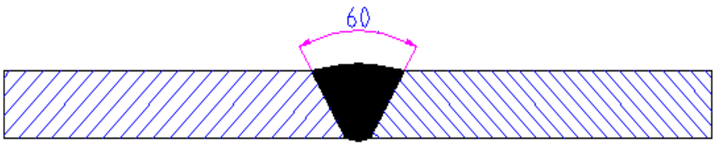
② Butt welds joints:

It is obtained by putting the edges of the two pieces together, there types are:

A – **Square Butt weld**

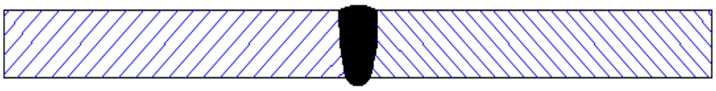


B – **Single V Butt weld**



C – **Double V Butt weld**

D – **U Butt weld**

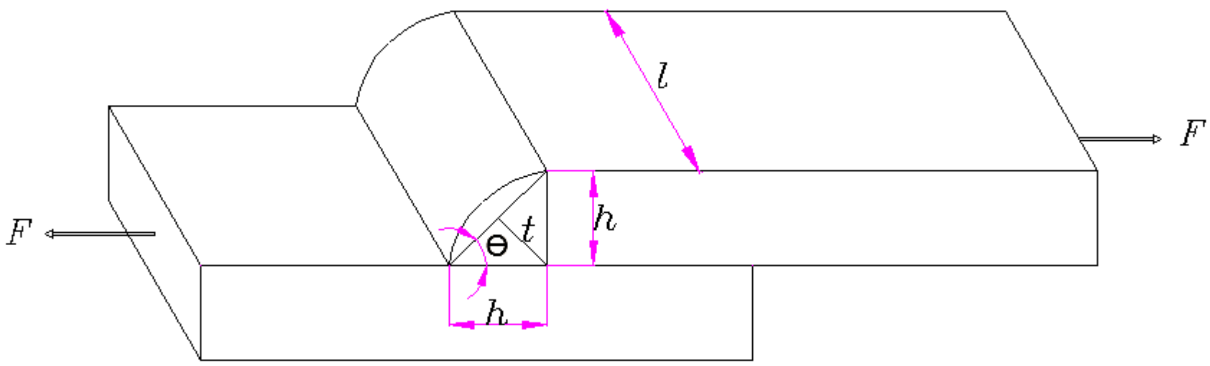


Strength of welds:

① *strength of lap welds joints*

A- strength of transverse fillet weld

I. For single transverse fillet weld



$$\sin\theta = \frac{t}{h}$$

$$t = h\cdot\sin\theta = h\cdot\sin 45$$

$$\sigma_t = \frac{F}{A_w}$$

Where: $A_w = t\cdot l$

$$\sigma_t = \frac{F}{l\cdot h\cdot\sin 45}$$

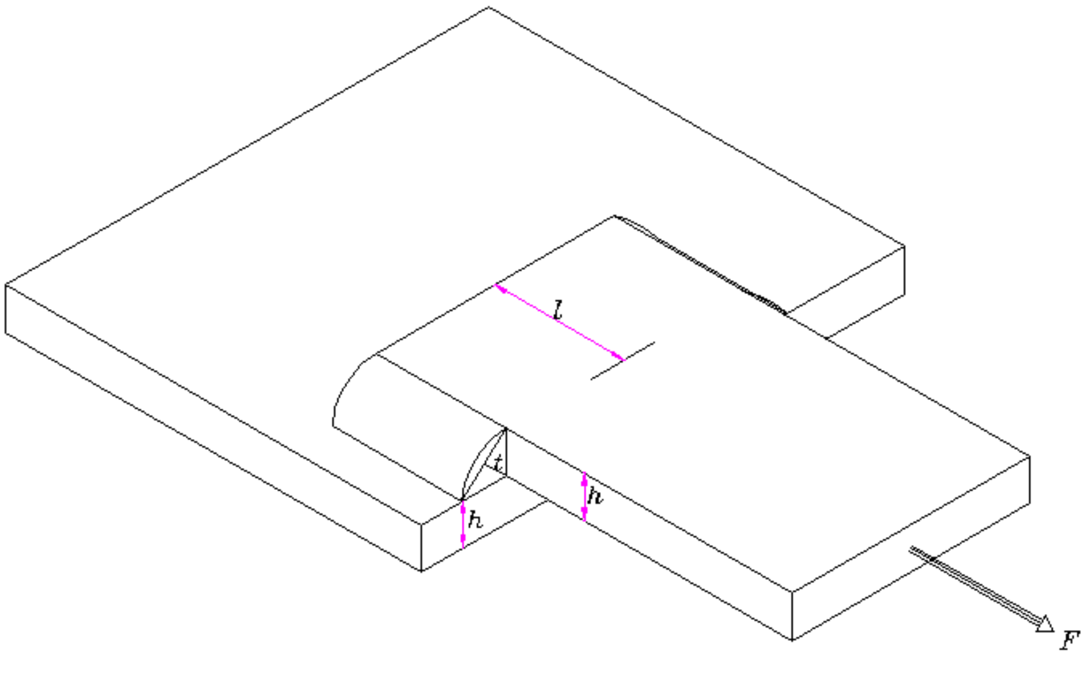
II. For double transverse fillet weld

$$\sigma_t = \frac{F}{2\cdot A_w}$$

$$\sigma_t = \frac{F}{2\cdot l\cdot h\cdot\sin 45}$$

B- strength of Parallel fillet weld

It is designed on the base of shear stress



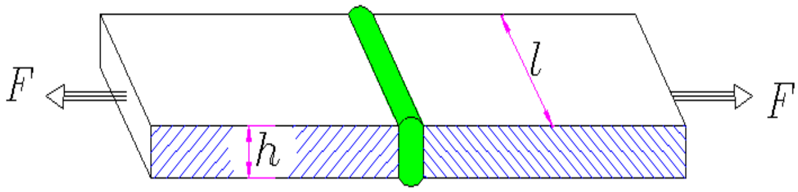
$$\tau = \frac{F}{A_w}$$

$$\tau = \frac{F}{2 \cdot l \cdot h \cdot \sin 45}$$

2 strength of butt welds joints

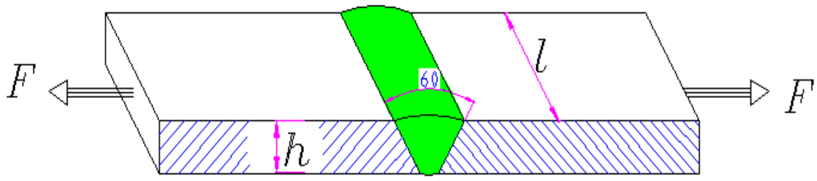
A – for Square Butt weld

$$\sigma_t = \frac{F}{A_w} = \frac{F}{h \cdot l}$$



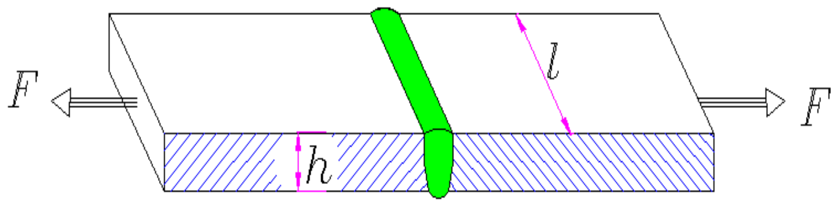
B – for Single V Butt weld

$$\sigma_t = \frac{F}{A_w} = \frac{F}{h \cdot l}$$



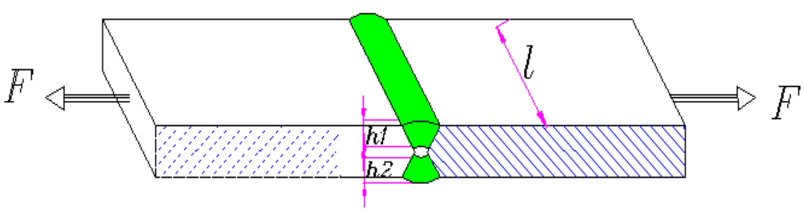
C – for U Butt weld

$$\sigma_t = \frac{F}{A_w} = \frac{F}{h \cdot l}$$



D – for Double V Butt weld

$$\sigma_t = \frac{F}{A_w} = \frac{F}{(h_1 + h_2) \cdot l}$$

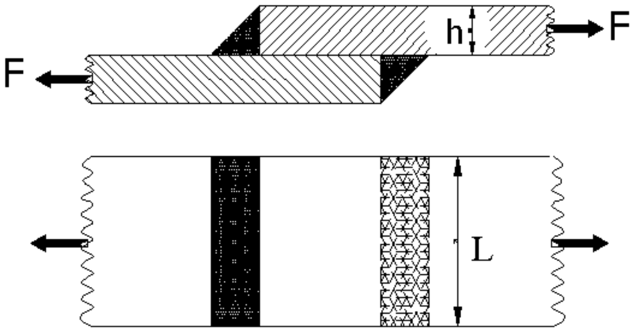


Example (1)

*Two plates are jointed by double transverse fillet (lap) weld. if the allowable tension stress for weld material **(σ_t =105 N/mm^2),** plates thickness **(6 mm)** and the total length of weld is **(l =100 mm)** , determine the maximum load of weld joint.*

Solution:

$$\sigma_t = \frac{F}{A_w} = \frac{F}{2 \cdot l \cdot h \cdot \sin 45}$$



$$F = 2 \cdot l \cdot h \cdot \sin 45 \cdot \sigma_t$$

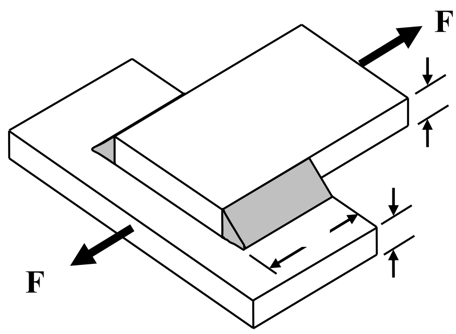
$$F = 2 \times 100 \times 6 \times \sin 45 \times 105 = 89095 N = 89.1 KN$$

Example (2)

*Determine the length of the parallel fillet lap welding required for joining two plates with thickness **(10 mm)**, if the allowable load is **(F=40 KN)** and the shear stress of weld material **(τ =80 MN/m^2).***

Solution:

$$\tau = \frac{F}{A_w}$$



$$\tau = \frac{F}{2 \cdot l \cdot h \cdot \sin 45}$$

$$l = \frac{F}{2 \cdot h \cdot \sin 45 \cdot \tau} = \frac{40 \times 10^3}{2 \times 0.01 \times \sin 45 \times 80 \times 10^6}$$

$$l = 0.0354m = 35.4mm$$

Example (3)

*A spherical gas tank is made of **(1 cm)** steel plate hemispheres butt weld together the tank is **(1500 cm)** in diameter. Determine the allowable internal pressure to which the tank may be subjected if the permissible stress be limited to **(84 MN/m²)***

Solution:

$$A_w = l \cdot h = \pi \cdot d \cdot h = \pi \times \frac{1500}{100} \times \frac{1}{100} = 0.47m^2$$

Bursting load resisted by the weld

$$\sigma_t = \frac{F}{A_w}$$

$$F = \sigma_t \cdot A_w = 84 \times 10^6 \times 0.47 = 39480000\,.N = 39.48 \times 10^6 N$$

Let the gas pressure is P

$$P = \frac{F_{bur}}{A} \Longrightarrow F_{bur} = P \cdot A = P \times \frac{\pi}{4} \times (15)^2$$

$$\frac{\pi}{4} \times (15)^2 \times P = 39.48 \times 10^6$$

$$P = \frac{39.48 \times 10^6}{\frac{\pi}{4} \times (15)^2} = 223411 \frac{N}{m^2}$$

Screws

Types of screws → *Fastening screw (bolts)*

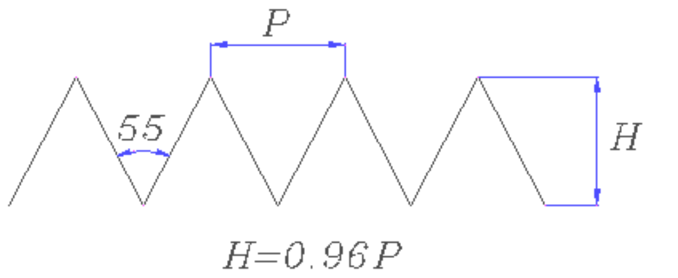
→ *Power screw*

Fastening screw (joints)

Types of screw teeth

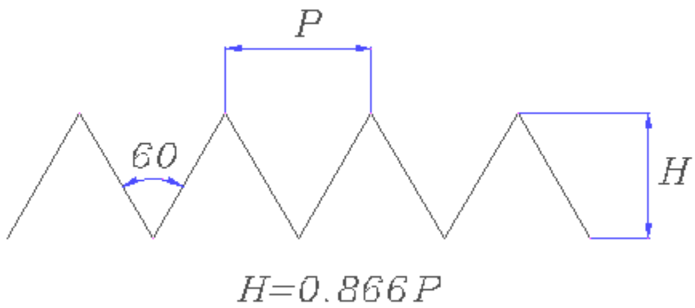
1- British standard

Angle of tooth =55°



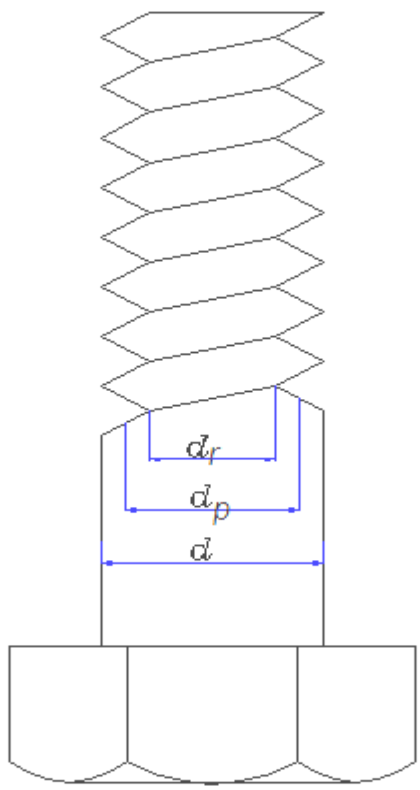
2-American national standard

Angle of tooth =60°



Definitions

d_i : inside diameter :root diameter (d_r)
d_p: pitch diameter.
d : outside diameter : nominal diameter.



Design of bolt:

The bolts are subjected to these stresses

1- tensile stress

2- shear stress

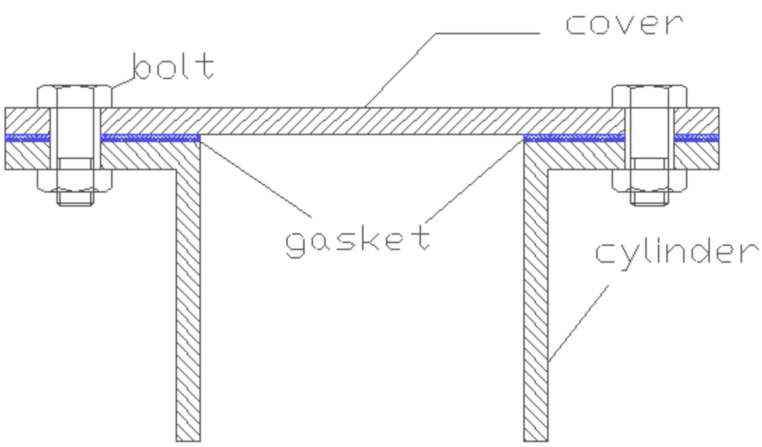
3- torsion stress

but in practice we cannot determine all these stresses with high accuracy so we calculate the screw only one the base of direct tensile stress and take the appropriate factor of safety

$$S.F = (2 \rightarrow 2.5)$$

Design bolts for tension :

The total force act on the screw can be calculated by



$$F_t = F_i + K \cdot F_e$$

$$F_i = 2840 \times d$$

$$F_e = P \cdot A$$

F_i: initial tension on bolt in ***(N)***

d: nominal diameter of bolt ***(mm)***

K: stiffness constant (calculated from table (1)

F_e: external force ***(N)***

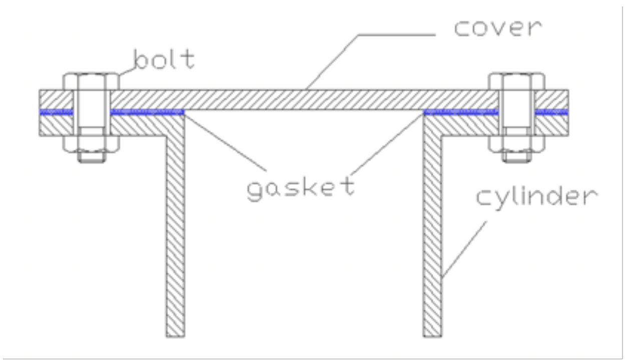
P: pressure inside the cylinder ***(N/m^2)***

A: cross section area of cylinder **(m^2)**

the value of "K" determined from table(1) and depend on the joint type and gasket type.

Table (1)

	Type of joint and gasket	***Value of "K"***
1	*Soft gasket with stud*	1
2	*Soft gasket with through bolt*	0.75
3	*Asbestos gasket with through bolt*	0.6
4	*Copper gasket with through bolt*	0.5
5	*Hard Copper gasket with through bolt*	0.25
6	*With out gasket by using stud*	0.1
7	*With out gasket by using through bolt*	0



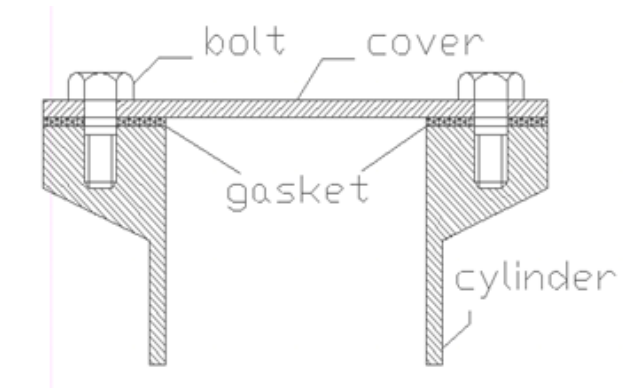


Table (2) size of bolts

size of bolt d(mm)	*pitch p(mm)*	*Root section area (mm^2)*
M1.6	0.35	1.27
M2	0.4	2.07
M2.5	0.45	3.39
M3	0.5	5.03
M4	0.8	14.2
M6	1	20.1
M8	1.25	36.6
M10	1.5	58
M12	1.75	84.3
M16	2	157
M18	2.5	192
M20	2.5	245
M22	2.5	303
M24	3	353
M27	3	459
M30	3.5	561
M33	3.5	694
M36	4	817
M39	4	976
M42	4.5	1120
M48	5	1470

Tensile stress act on the screw

$$\sigma_t = \frac{F_t}{A_r}$$

Design bolts for shear:

Shear stress act on the screw

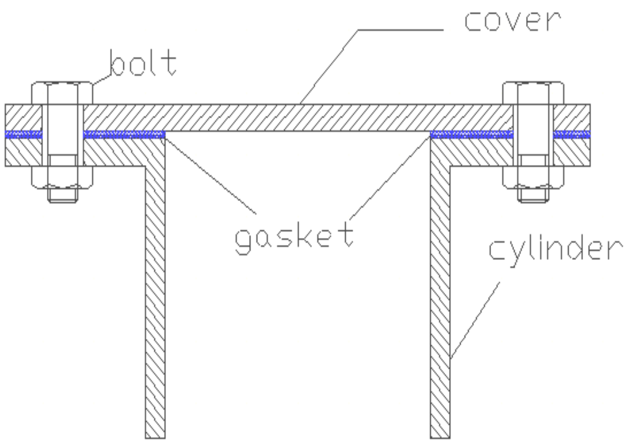
$$\tau = \frac{F}{A_r}$$

F: shear force **(N)**

A_r: root section area for bolt determine from table (2) according to the size of bolt.

Example (1)

Cylinder cover of internal combustion engine is jointed by 10 through bolts. if the diameter of the cylinder ***(30 cm)****, gases pressure* ***(80 KN/m²)*** *tension stress for the bolt material is* ***(100 MN/m²). Determine the size of bolts.***



Solution:

$$F_e = P \cdot A = 80000 \times \frac{\pi}{4}(0.3)^2 = 5655.N$$

$$External\ force\ for\ one\ bolt = \frac{5655}{10} = 565.5N$$

From table (1) K=0.75

$$F_i = 2840 \times d$$

$$F_t = F_i + K \cdot F_e = 2840 \times d + 0.75 \times 565.5$$

$$F_t = 2840 \times d + 424$$

$$\sigma_t = \frac{F_t}{A_r} \Longrightarrow 100 = \frac{2840 \times d + 424}{\frac{\pi}{4} \times (d_r)^2}$$

$$78.5(d_r)^2 - 2840d_r - 424 = 0$$

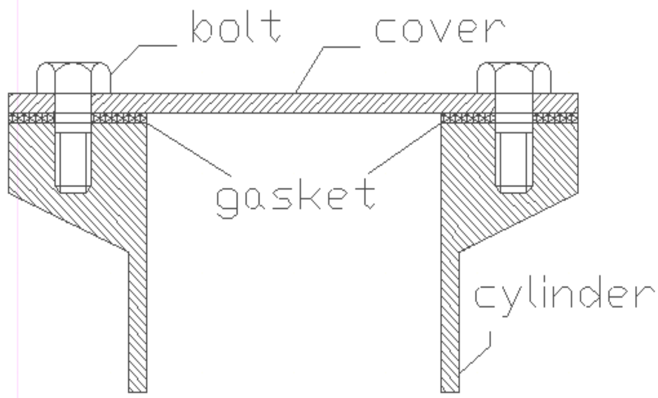
$$(d_r)^2 - 36.2d_r - 5.4 = 0$$

$$d_r = 36.3mm \qquad A_r = \frac{\pi}{4}\times(d_r)^2 = 1037.7mm^2$$

From table (2) the size of bolt is M_{42}

Example (2)

*The cylinder head of steam engine is held in position by **(12)** studs. the cylinder bore is **(500 mm)** and the max. pressure is **(12 kg/cm²)**. if the tensile stress for stud is **(1000 kg/cm²)** and the stiffness coefficient **(K=0.25)**. Determine the size of the studs.*



Assume the initial force. $F_i = 2840\times d$

Solution:

D = 500 mm

*P =12*10/100=1.2 N/mm²*

$$\sigma_t = \frac{1000\times 10}{100} = 100\frac{N}{mm^2}$$

$$F_e = P\cdot A = 1.2\times\frac{\pi}{4}(500)^2 = 235619.4N$$

$$F_e \quad per \quad stud = \frac{235619.4}{12} = 19635N$$

$$F_t = F_i + K\cdot F_e = 2840\times d + 0.25\times(19635)$$

$$F_t = 2840\times d + 4908.7$$

$$\sigma_t = \frac{F_t}{A_r} \Longrightarrow 100 = \frac{2840 \times d + 4908.7}{\frac{\pi}{4} \times (d_r)^2}$$

$$78.5(d_r)^2 - 2840 \times d_r - 4908.7 = 0$$

$$(d_r)^2 - 36.2 d_r - 62.5 = 0$$

$$d_r = 37.9mm$$

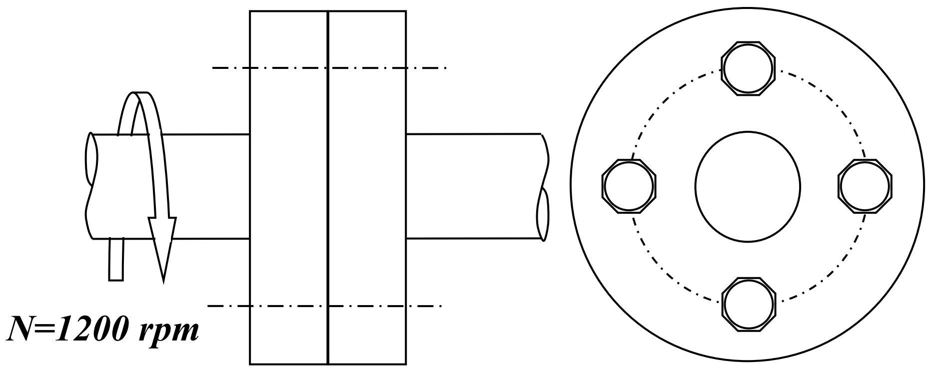
$$d_r = -1.7mm$$

$$\therefore d_r = 37.9mm \qquad A_r = \frac{\pi}{4} \times (d_r)^2 = 1128.2mm^2$$

From table (2) the size of bolt is M_{42}

Example (3)

*Rigid coupling translate power **(15 KW)** and turn with speed **(1200 rpm)** as shown in fig. if the driver and driven parts are fastened by four bolts arranged on the circumference of circle with diameter of* **(12 cm)** and *the shear stress for every bolt **(τ = 15 MN/m²)**, determine the size of the bolts.*



Solution:

Angular speed $w = \frac{2 \cdot \pi \cdot N}{60} = \frac{2 \times \pi \times 1200}{60}$

$$w = 125.7 \frac{rad}{\sec}$$

$$Power = P = T \cdot w$$

$$15000 = T \times 125.7$$
$$T = 119.4.N \cdot m$$

$$T = F \cdot R \cdot n$$

$$119.4 = F \times 0.06 \times 4 \quad \Longrightarrow \quad F = 497.4N$$

$$\tau = \frac{F}{A_r}$$

$$15 \times 10^6 = \frac{497.4}{A_r}$$
$$A_r = 3.315 \times 10^{-5} m^2$$
$$A_r = 33.15 mm^2$$

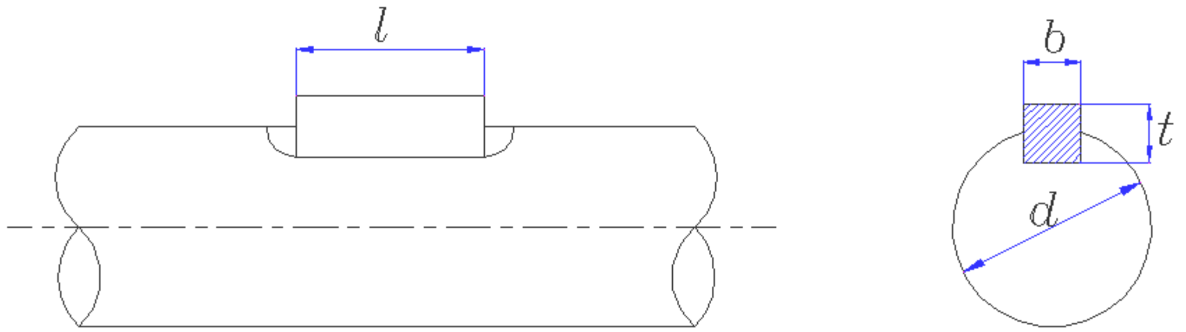
From table (2) we find   d = 8 mm   so the size of bolt is M_8

Keys:

The function of a key is to prevent relative rotation of a shaft and the hub [such as gear, pulley] it is non permanent joint.

Types of key:

① **sunk key:** *in this type half of the key is sunken in the shaft and the other in the hub [gear or pulley]*



There types are:

A - Rectangular sunk key:

Sunk key with rectangular cross section may be of uniform cross section or may be tapered by 1:100 for easy and stable connection

Where:

b: width

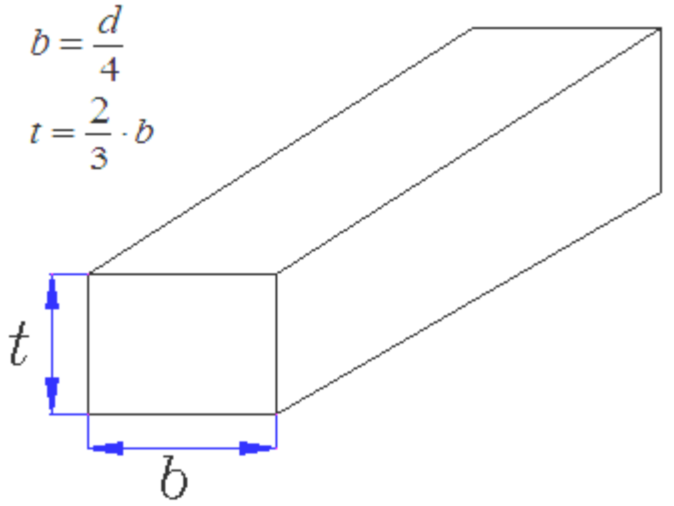
d: diameter of shaft

t: thickness

l: length of key

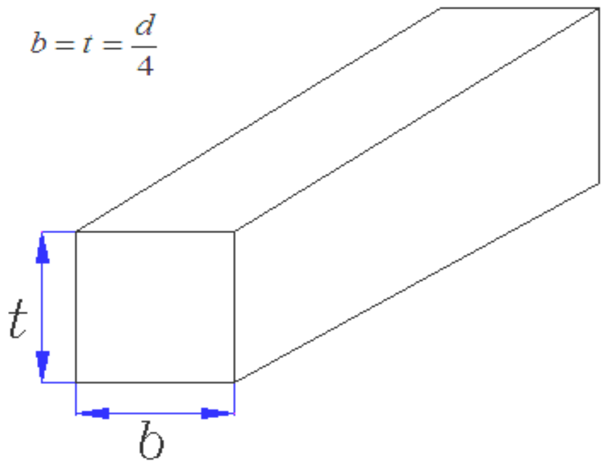
$$b = \frac{d}{4}$$

$$t = \frac{2}{3} \cdot b$$



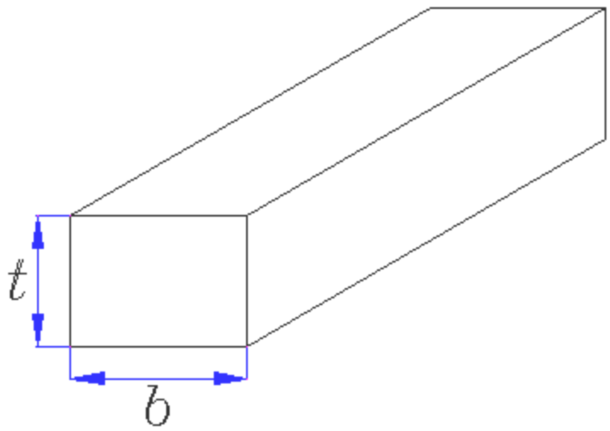
B - Square sunk key:

This type has the same width and depth



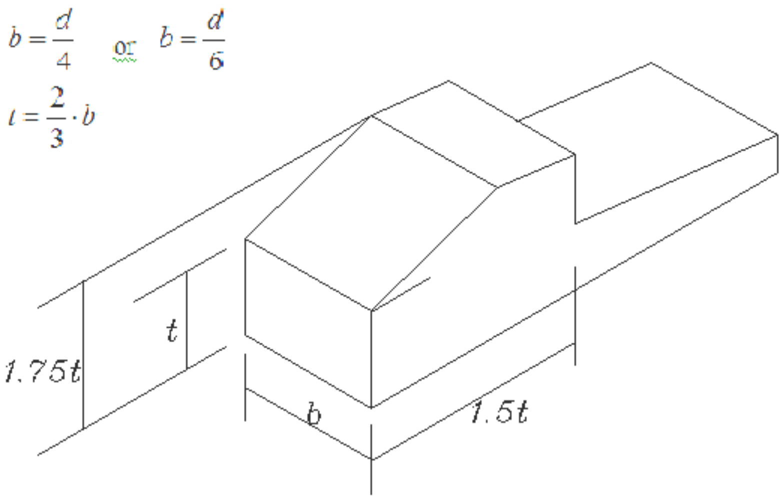
C - Parallel sunk key:

The cross section of this type may be rectangular or square and is used for gear and pulley which are turn and slide over the shaft in the same time, this type is untapered.



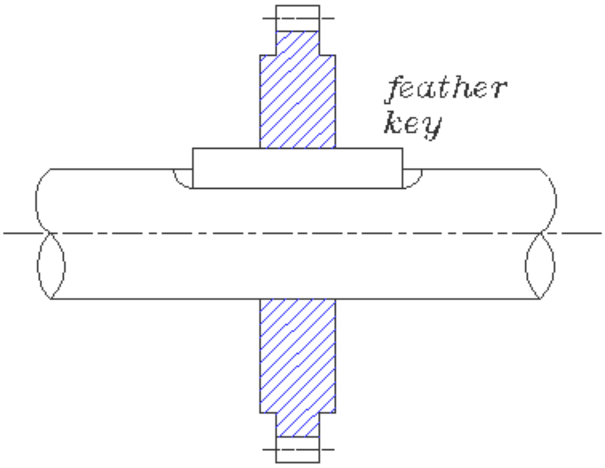
D - Gib head sunk key:

It is a tapered key with rectangular cross section area and has gib head for easy installation.



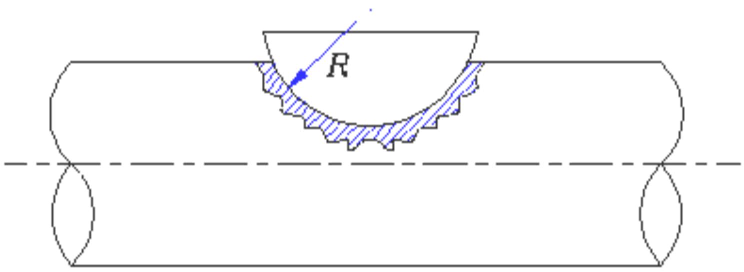
E - Feathers key:

This is special type used to transmit turning moment and allow for axial motion it joints on the shaft or pulley by tapered pin or it has two heads



F - Wood ruff sunk key:

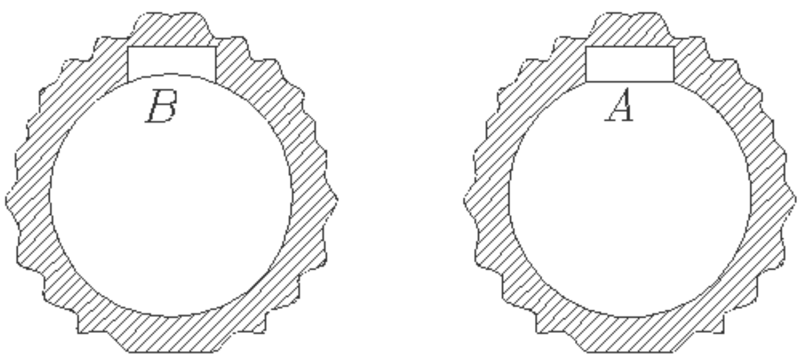
It has the form of semi-circle and requires a special side milling cutter to form the key seat. disadvantage of this key is weaking the shaft and it is used in automobile joints.



(2) Saddle keys:

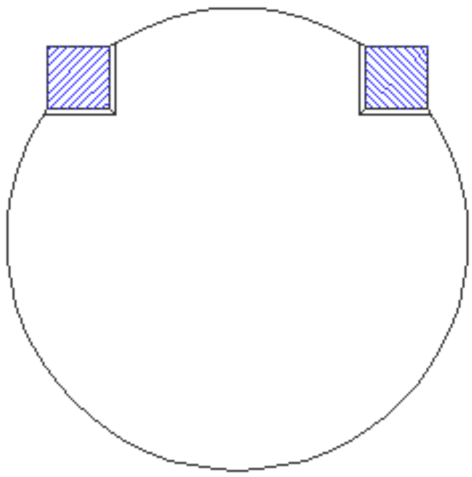
A - Flat saddle key: *used for light load.*

B - Hollow saddle key



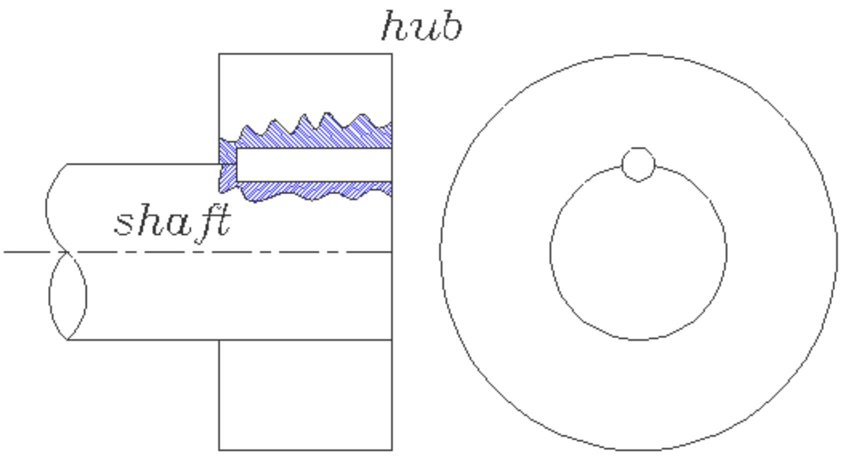
③ tangent keys:

Used for heavy duty and consists of two key which are fixed on the shaft with right angle every key bear the turning moment in one direction.



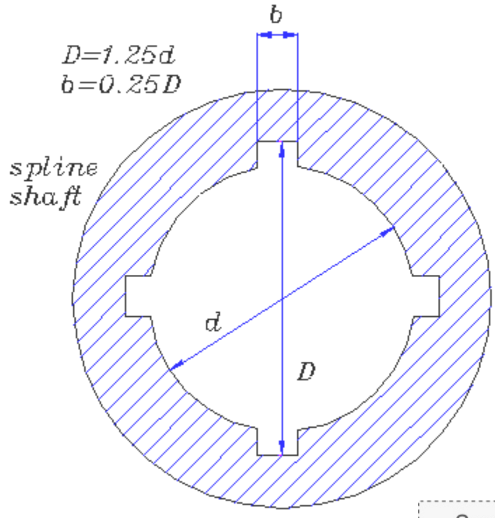
④ Round key :

This type has a circular cross section area and is fixed in a hole which is drilled between shaft and hub. this type is used for light duty.



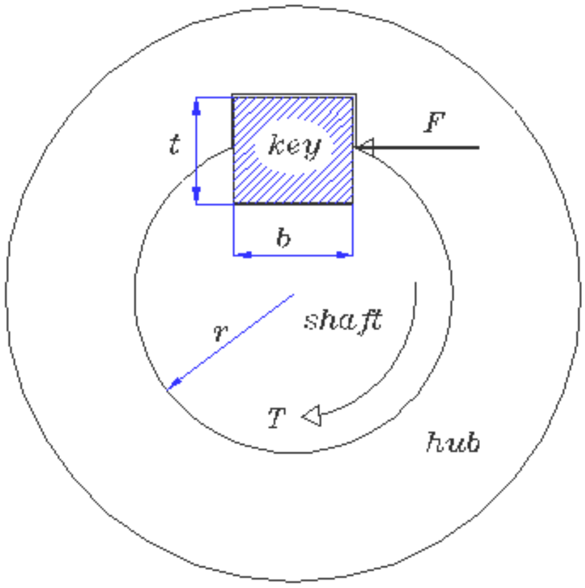
⑤ Splines :

Used for heavy duty. It is composed of a splined shaft formed by milling and mating hub with internal splines.



Design of sunk key:

The key is subjected to shear stress and bearing stress due to the tangential force.



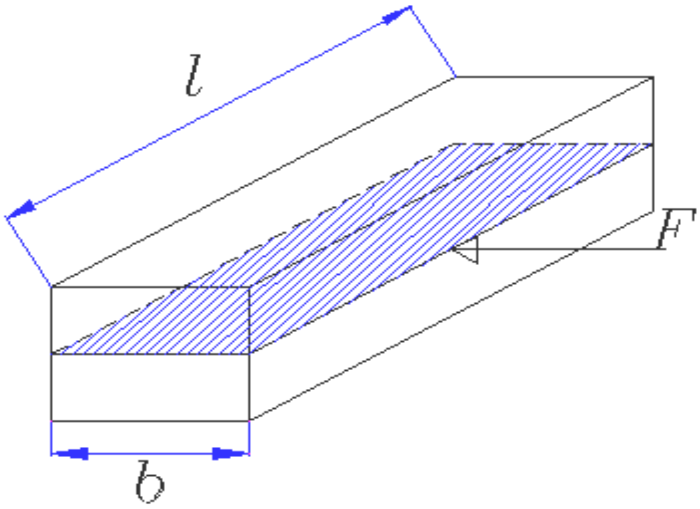
1 - According to shear stress:

$$\tau = \frac{F}{A} = \frac{F}{b \cdot l}$$

$$\because T = F \cdot \frac{d}{2}$$

$$\therefore F = \frac{2 \cdot T}{d}$$

$$\tau = \frac{2 \cdot T}{d \cdot b \cdot l}$$

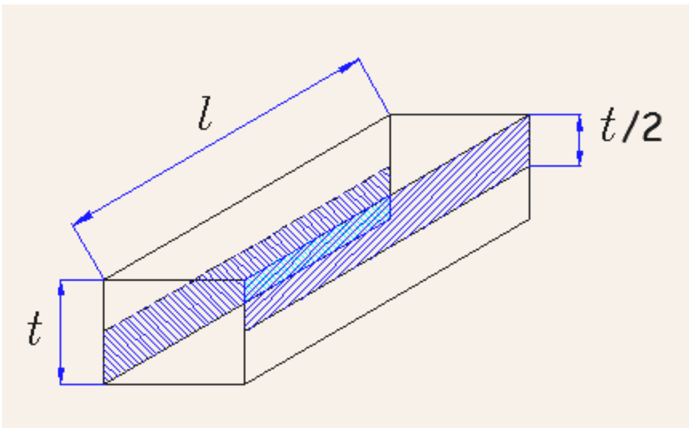


2 - According to bearing or (crushing) stress:

$$\sigma_{bearing} = \frac{F}{A_{proj}} = \frac{F}{\frac{t}{2} \cdot l}$$

$$\because T = F \cdot \frac{d}{2} \qquad \therefore F = \frac{2 \cdot T}{d}$$

$$\sigma_{bearing} = \frac{2 \cdot T}{\frac{t}{2} \cdot l \cdot d}$$



Example (1)

*Determine the dimensions of the rectangular sunk key. If the diameter of the shaft **(d =40 mm),** transmitted force **(F = 20 KN)**, thickness of key **(t= 10 mm)**, permissible bearing stress **($\sigma_{bearing}$ = 250 MN/m^2)*** and the shear stress ***(τ =104 MN/m^2)***.

Solution :

$$\sigma_{bearing} = 250 \times 10^6 \frac{N}{m^2}$$

$$\tau = 104 \times 10^6 \frac{N}{m^2}$$

$$F = 20 \times 10^3 N$$

$$t = \frac{10}{1000} = 0.01m$$

$$\sigma_{bearing} = \frac{F}{A_{proj}} = \frac{F}{\frac{t}{2} \cdot l}$$

$$250 \times 10^6 = \frac{20 \times 10^3}{\frac{0.01}{2} \times l} \Longrightarrow l = 0.016m$$

$$\therefore l = 16mm$$

$$\tau = \frac{F}{A} = \frac{F}{b \cdot l}$$

$$104 \times 10^6 = \frac{20 \times 10^3}{b \times 0.016} \Longrightarrow b = 0.012m$$

$$\therefore b = 12mm$$

Example (2)

A rectangular sunk key with dimension ***L*b*t=75*14*10mm*** *is required to transmit a torque* ***(T = 1200 N.m)*** *from a solid shaft of diameter* ***(d = 50 mm)*** *determine weather the length is sufficient or not , if the permissible shear stress and crushing stress are* ($\tau = 56\ MN/m^2$), ($\sigma_{crush} = 168\ MN/m^2$).

Solution :

$$l = \frac{75}{1000} = 0.075m \quad , \quad b = \frac{14}{1000} = 0.014m \quad , \quad t = \frac{10}{1000} = 0.01m$$

$$T = 1200\, N \cdot m \quad , \quad d = \frac{50}{1000} = 0.05m \quad , \quad \tau_{per} = 56 \times 10^6 \frac{N}{m^2}$$

$$\sigma_{crush)per} = 168 \times 10^6 \frac{N}{m^2}$$

$$\tau_{cal} = \frac{2 \cdot T}{d \cdot b \cdot l} = \frac{2 \times 1200}{0.05 \times 0.014 \times 0.075} = 45714286 \frac{N}{m^2}$$

$$\tau_{cal} = 45.7 \frac{MN}{m^2} \quad < \quad \tau_{per} = 56 \frac{MN}{m^2} \quad \text{O.K}$$

$$\sigma_{crush)cal} = \frac{2 \cdot T}{\frac{t}{2} \cdot l \cdot d} = \frac{2 \times 1200}{\frac{0.01}{2} \times 0.075 \times 0.05} = 128000000 \frac{N}{m^2}$$

$$\sigma_{crush)cal} = 128 \frac{MN}{m^2} \quad < \quad \sigma_{crush)per} = 168 \frac{MN}{m^2} \quad \text{O.K}$$

the length of key is sufficient

Example (3)

*A belt pulley transmitting power is secured to a steel shaft **(50 mm)** diameter. The key provided has a width of **(18mm)** and a thickness of **(16mm)** and is **(125mm)** long. the material of the key allows a stress of **(400kg/cm²)** in shear and **(950kg/cm²)** in bearing. what h.p can be transmitted by the pulley when running at **(200rpm).***

Solution:

$$d = \frac{50}{1000} = 0.05m \quad , \quad b = \frac{18}{1000} = 0.018m \quad , \quad t = \frac{16}{1000} = 0.016m$$

$$l = \frac{125}{1000} = 0.125m \quad , \quad \tau = 400\frac{kg}{cm^2} = \frac{400 \times 10}{10^{-4}} = 40000000\frac{N}{m^2}$$

$$\sigma_{bearing} = 950\frac{kg}{cm^2} = \frac{950 \times 10}{10^{-4}} = 95000000\frac{N}{m^2} \quad , \quad N = 200.rpm$$

$$\tau = \frac{F}{A} = \frac{F}{b \cdot l} \Longrightarrow 40 \times 10^6 = \frac{F}{0.018 \times 0.125}$$

$$F = 90000N$$

$$\sigma_{bearing} = \frac{F}{A_{proj}} = \frac{F}{\frac{t}{2} \cdot l} \Longrightarrow 95 \times 10^6 = \frac{F}{\frac{0.016}{2} \times 0.125}$$

$$F = 95000N$$

$$\therefore F = 90000N$$

$$T = F \cdot \frac{d}{2} = 90000 \times \frac{0.05}{2} = 2250N \cdot m$$

$$power = T \cdot w = 2250 \times \left(\frac{2 \times \pi \times 200}{60}\right) = 47124.watt$$

$$power = 47124 \div 746 = 63.2.h.p$$

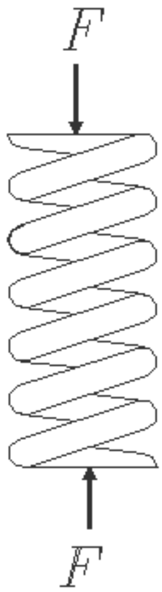
Springs:

It defined as an elastic body whose function is to deflect under load and when the load is released it return to its original shape.

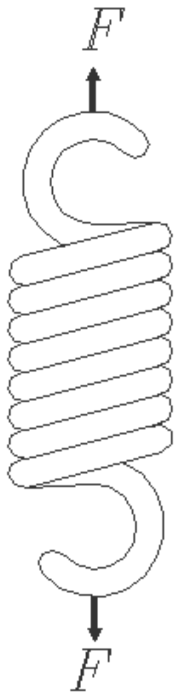
Types of springs:

(I) - Helical spring:

It is made of wire coiled in to helical form and is subjected to tensile force or compressive force



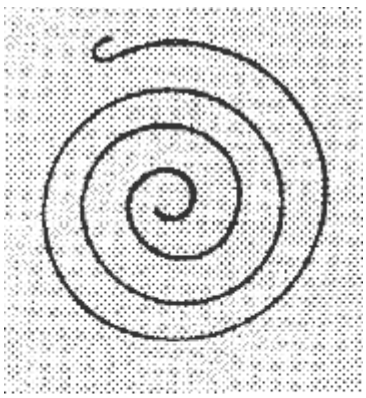
Compression helical spring



Tension helical spring

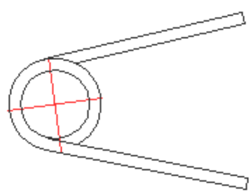
(II)-Helical torsion spring:

Also, it is made of wire coiled into a helical form and subjected to torsion moment.



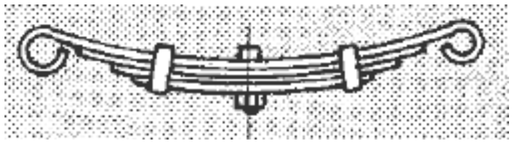
(III) -Spiral springs:

It consists of flat strip wound in the form of spiral and is subjected to torsion moment.



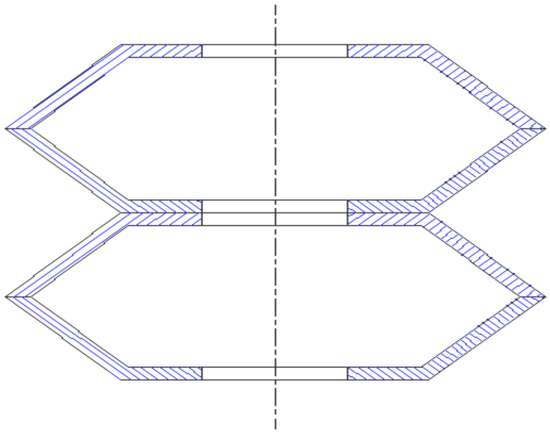
IV -Leaf spring:

It composed of flat bars of varying lengths clamped to gather and are subjected to load.

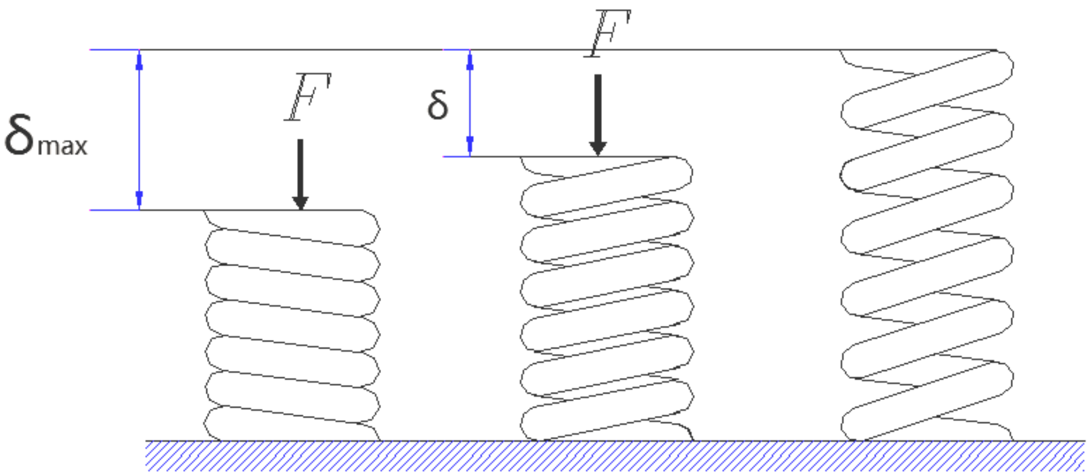


V -Belleville spring:

They are composed of coned discs and are subjected to compressive force.



Definition of helical spring:



1-Solid length: L_s

It is the length of spring when it is compressed to form hollow cylinder

$$L_s = n \cdot d$$

Where *n: number of active coils*
d: diameter of spring wire

2-Free length: L_f

Length of spring when it is not subjected to external force.

$$L_f = L_s + \delta_{max} + \varepsilon$$

$$L_f = n \cdot d + \delta_{max} + (n-1) \times 0.1$$

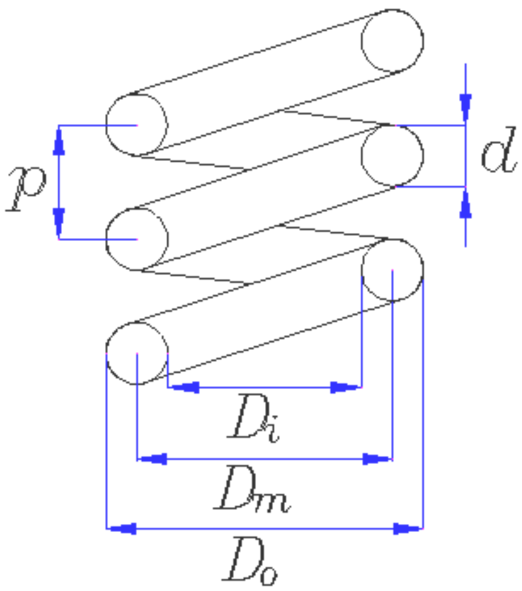
Where δ : *deflection of spring*

δ_{max} : *max. deflection of spring*

ε : *spring end coils*

3- Spring index (C):

It is the ratio between the mean diameter of coil spring and the diameter of spring wire



$$C = \frac{D_m}{d}$$

$$D_m = D_i + d$$
$$D_m = D_o - d$$

4- Spring rate (stiffness) (K):

It is defined as a force which is required to extend or expand the spring to one unit length.

$$K = \frac{F}{\delta} \quad \frac{N}{m}$$

Where F : *external force (N)*

δ : *Deflection of spring (m)*

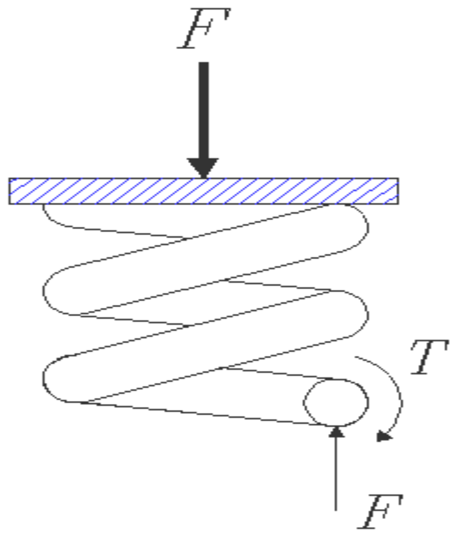
5-pitch of coil spring:

It is the distance between any two coils when the spring is free

$$p = \frac{L_f}{(n-1)}$$

Design of helical spring:

Helical spring is designed on the base of shear stress due to torsion and direct shear (neglected the curvature effect)



I –Direct shear stress

$$\tau_d = \frac{F}{\frac{\pi}{4} \cdot d^2} = \frac{4 \cdot F}{\pi \cdot d^2}$$

II-Torsion shear stress

$$\tau_t = \frac{T \cdot r}{J} = \frac{F \cdot \frac{D_m}{2} \cdot \frac{d}{2}}{\frac{\pi}{32} \cdot d^4} = \frac{8 \cdot F \cdot D_m}{\pi \cdot d^3}$$

$$\tau = \tau_d + \tau_t = \frac{4 \cdot F}{\pi \cdot d^2} + \frac{8 \cdot F \cdot D_m}{\pi \cdot d^3}$$

$$\tau = \frac{8 \cdot F \cdot D_m}{\pi \cdot d^3} \left(\frac{d}{2 \cdot D_m} + 1 \right) \qquad \because C = \frac{D_m}{d}$$

$$\tau = \frac{8 \cdot F \cdot D_m}{\pi \cdot d^3} \left(\frac{1}{2 \cdot C} + 1 \right)$$

By taking curvature effect

$$\tau = \frac{8 \cdot k \cdot F \cdot D_m}{\pi \cdot d^3} \qquad or \qquad \tau = \frac{8 \cdot k \cdot F \cdot C}{\pi \cdot d^2}$$

$$k = \frac{4 \cdot C - 1}{4 \cdot C - 4} + \frac{0.615}{C}$$

Calculation of spring deflection

1-Angular deflection: θ

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G \cdot \theta}{L} \qquad \theta = \frac{T \cdot L}{J \cdot G}$$

$$\because T = F \cdot \frac{D_m}{2}$$

$$J = \frac{\pi}{32} \cdot d^4$$

$$L = \pi \cdot D_m \cdot n$$

$$\theta = \frac{F \cdot \frac{D_m}{2} \cdot \pi \cdot D_m \cdot n}{\frac{\pi}{32} \cdot d^4 \cdot G} = \frac{16 \cdot F \cdot D_m^2 \cdot n}{d^4 \cdot G}$$

2-Axial deflection: δ

$$\delta = \theta \cdot \frac{D_m}{2} = \frac{16 \cdot F \cdot D_m^2 \cdot n}{d^4 \cdot G} \times \frac{D_m}{2}$$

$$\delta = \frac{8 \cdot F \cdot D_m^3 \cdot n}{d^4 \cdot G}$$

$$\delta = \frac{8 \cdot F \cdot C^3 \cdot n}{d \cdot G}$$

Example (1)

*Helical spring with outside diameter **(D_0=7.5cm)** and manufactured from wire with diameter **(d = 6mm)** if the shear stress of wire material **(τ = 350 MN/m²)** and the modulus of rigidity **(G = 84 GN/m²)**.determine the external (axial) load and spring (axial) deflection per coil .*

Solution:

$$D_o = 7.5cm = \frac{7.5}{100} = 0.075m \quad , \quad d = 6mm = \frac{6}{1000} = 0.006m$$

$$\tau = 350 \times 10^6 \frac{N}{m^2} \quad , \quad G = 84 \times 10^9 \frac{N}{m^2}$$

$$D_m = D_o - d = 0.075 - 0.006 = 0.069m$$

$$C = \frac{D_m}{d} = \frac{0.069}{0.006} = 11.5$$

$$k = \frac{4 \cdot C - 1}{4 \cdot C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.125$$

$$\tau = \frac{8 \cdot k \cdot F \cdot C}{\pi \cdot d^2} \Longrightarrow F = \frac{\tau \cdot \pi \cdot d^2}{8 \cdot k \cdot C}$$

$$F = \frac{350 \times 10^6 \times \pi \times (0.006)^2}{8 \times 1.125 \times 11.5} = 382.5N$$

$$\delta = \frac{8 \cdot F \cdot C^3 \cdot n}{d \cdot G}$$

$$\frac{\delta}{n}=\frac{8\cdot F\cdot C^3}{d\cdot G}=\frac{8\times 382.5\times(11.5)^3}{0.006\times 84\times 10^9}$$

$$\frac{\delta}{n}=0.00923m$$

Example (2)

Helical spring with mean diameter ***(D_m=25mm)*** *and manufactured from wire with diameter* ***(d=3mm).****if the shear stress of spring material* ***(τ =441 MN/m²)****, axial deflection* ***(δ=25mm)*** *and the modulus of rigidity* ***(G =86.2 GN/m²)****. Determine the external force and the number of active coils.*

Solution:

$$D_m=0.025m \quad , \quad d=0.003m \quad , \quad \tau=441\times10^6\frac{N}{m^2}$$

$$\delta=0.025m \quad , \quad G=86.2\times10^9\frac{N}{m^2}$$

$$C=\frac{D_m}{d}=\frac{0.025}{0.003}=8.3$$

$$k=\frac{4\cdot C-1}{4\cdot C-4}+\frac{0.615}{C}=\frac{4\times8.3-1}{4\times8.3-4}+\frac{0.615}{8.3}=1.177$$

$$\tau=\frac{8\cdot k\cdot F\cdot C}{\pi\cdot d^2} \Longrightarrow F=\frac{\tau\cdot\pi\cdot d^2}{8\cdot k\cdot C}$$

$$F=\frac{441\times10^6\times\pi\times(0.003)^2}{8\times1.177\times8.3}=159.5N$$

$$\delta=\frac{8\cdot F\cdot C^3\cdot n}{d\cdot G} \Longrightarrow n=\frac{\delta\cdot d\cdot G}{8\cdot F\cdot C^3}$$

$$n = \frac{0.025 \times 0.003 \times 86.2 \times 10^9}{8 \times 159.5 \times (8.3)^3} = 8.9 \approx 9$$

Example (3)

*Design helical compression spring to carry load (F=1000N) and having axial deflection (δ =25mm) spring index (C = 5), shear stress **(τ =420 MN/m²)**, and the modulus of rigidity **(G =86.2 GN/m²)**.*

Solution:

$F = 1000 N$, $\delta = 0.025 m$, $C = 5$, $\tau = 420 \times 10^6 \frac{N}{m^2}$

$G = 84 \times 10^9 \frac{N}{m^2}$

1- Diameter of wire (d)

$$k = \frac{4 \cdot C - 1}{4 \cdot C - 4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

$$\tau = \frac{8 \cdot k \cdot F \cdot C}{\pi \cdot d^2} \Longrightarrow d = \sqrt{\frac{8 \cdot k \cdot F \cdot C}{\pi \cdot \tau}}$$

$$d = \sqrt{\frac{8 \times 1.31 \times 1000 \times 5}{\pi \times 420 \times 10^6}} = 0.0063 m = 6.3 mm$$

2- Outside and inside diameter D_o ,D_i

$$C = \frac{D_m}{d} \Longrightarrow D_m = C \cdot d = 5 \times 6.3 = 31.5 mm$$

$$D_o = D_m + d = 37.8 mm$$

$$D_i = D_m - d = 25.2 mm$$

3- Number of active coils n

$$\delta = \frac{8 \cdot F \cdot C^3 \cdot n}{d \quad \cdot G} \Longrightarrow n = \frac{\delta \cdot d \cdot G}{8 \cdot F \cdot C^3}$$

$$n = \frac{0.025 \times 0.0063 \times 84 \times 10^9}{8 \times 1000 \times (5)^3} = 13.23 \qquad \therefore n = 14$$

4-solid length L_s

$$L_s = n \cdot d = 14 \times 6.3 = 88.2mm$$

5-free length L_f

$$L_f = n \cdot d + \delta_{\max} + (n-1) \times 0.1$$

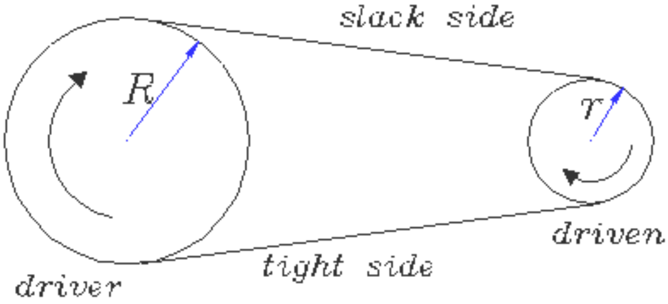
$$L_f = 14 \times 6.3 + 25 + (14-1) \times 0.1 = 114.5mm$$

6-pitch p

$$p = \frac{L_f}{(n-1)} = \frac{114.5}{13} = 8.81mm$$

Belts:

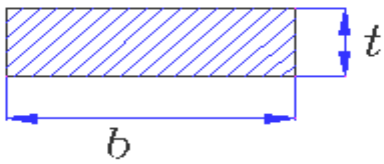
The belts are used to transmit the power from one shaft to another when the distance between the shaft axes is large and the angular velocity ratio of the driving and driven member is not constant or allow slip.



Types of Belts:

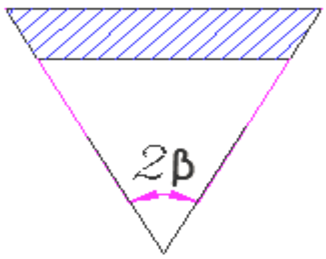
1) Flat belt:

It is used to transmit high power with low speed.



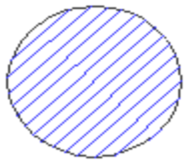
2) V- belt:

It is used to transmit power more than flat belt.



3) Circular belt (rope):

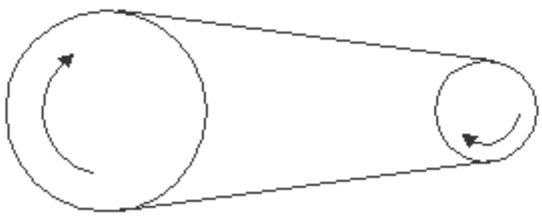
It is used to transmit high power with high speed.



Types of belt drives:

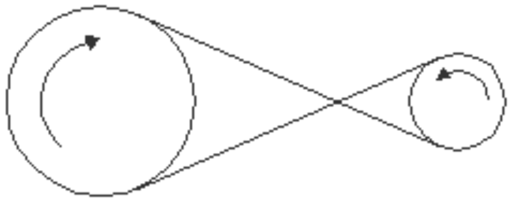
Ⓘ Open belt drive

This type is used to transmit power between two parallel axis and turning in same direction.



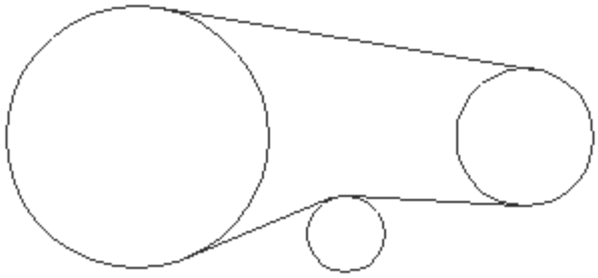
(II) *Cross belt drive*

It is used for transmitting power between two parallel axes, turn in opposite direction.



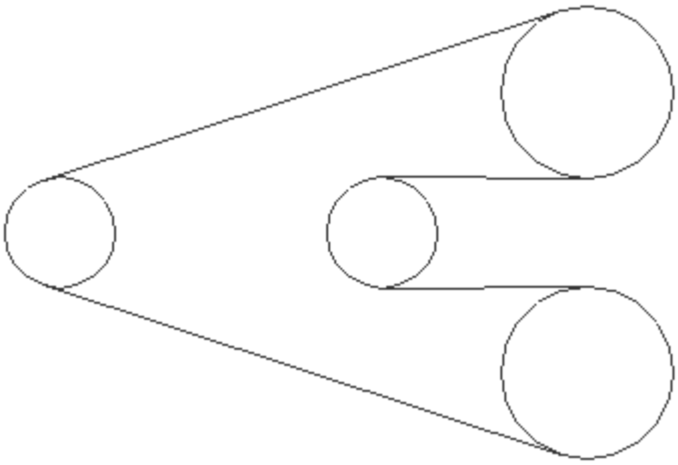
(III) *belt drive with idler pulley*

Idler pulley is used to increase are of contact and belt tension.



(IV) *Belt drive with many pulleys*

It is used to transmit power from one shaft to many shafts with one belt.



Design of belts:

1- Velocity ratio:

(A) *By neglecting slip*

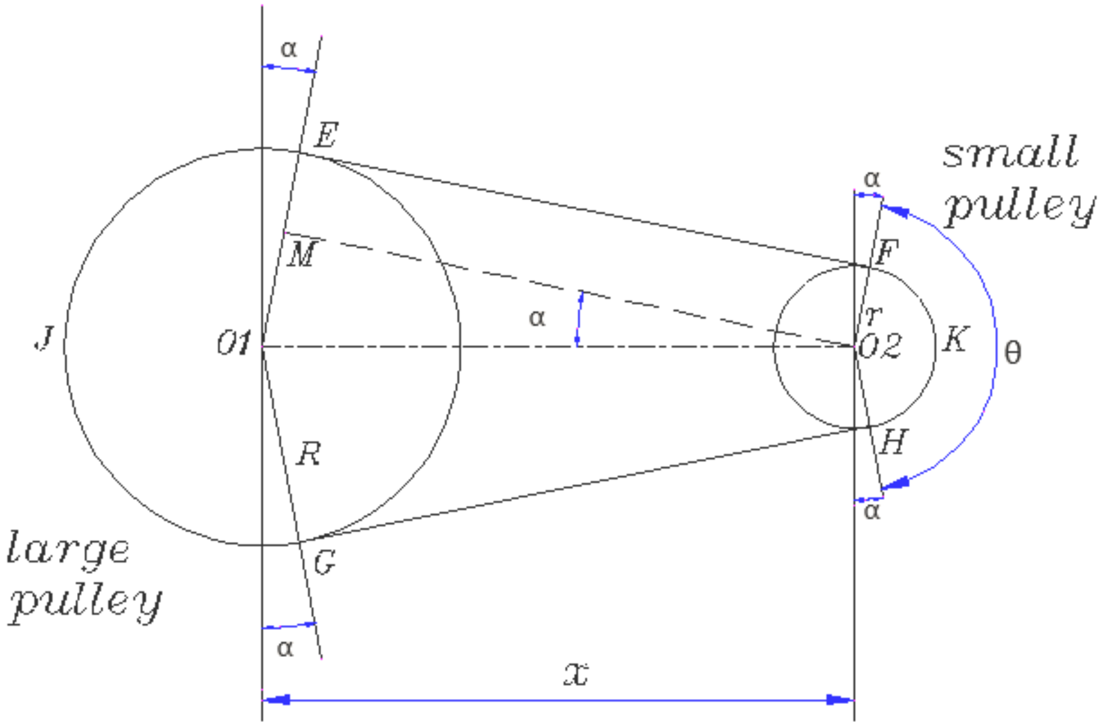
$$V_R = \frac{N_2}{N_1} = \frac{\omega_2}{\omega_1} = \frac{d_1}{d_2}$$

Ⓑ **By account slip**

$$V_R = \frac{N_2}{N_1} = \frac{\omega_2}{\omega_1} = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right)$$

2- Length of belt

Ⓐ***length of an open belt drive:***



Total length of belt is equal to.

$L = \overset{\frown}{GJE} + EF + \overset{\frown}{FKH} + HG$

$L = 2(\overset{\frown}{JE} + EF + \overset{\frown}{FK})$

from $\triangle MO_2O_1$ *we get*

$$\sin\alpha = \frac{R-r}{x} \Longrightarrow \alpha = \frac{R-r}{x}$$

$$Arc\widehat{JE} = R\left(\frac{\pi}{2}+\alpha\right)$$

$$Arc\widehat{FK} = r\left(\frac{\pi}{2}-\alpha\right)$$

$$EF = \sqrt{x^2-(R-r)^2} = x\sqrt{1-\left(\frac{R-r}{x}\right)^2}$$

Expanding by binomial theory

$$EF = x\left(1-\frac{1}{2}\left(\frac{R-r}{x}\right)^2+.....\right) \approx x-\frac{(R-r)^2}{2\cdot x}$$

$$L = 2\left(R\left(\frac{\pi}{2}+\alpha\right)+x-\frac{(R-r)^2}{2\cdot x}+r\left(\frac{\pi}{2}-\alpha\right)\right)$$

$$L = R\pi+2\cdot\alpha\cdot R+2\cdot x-\frac{(R-r)^2}{x}+r\pi-2\cdot\alpha\cdot r$$

$$L = \pi(R+r)+2\cdot x-\frac{(R-r)^2}{x}+2\cdot\alpha(R-r)$$

$$L = \pi(R+r)+2\cdot x-\frac{(R-r)^2}{x}+2\cdot\frac{(R-r)}{x}\cdot(R-r)$$

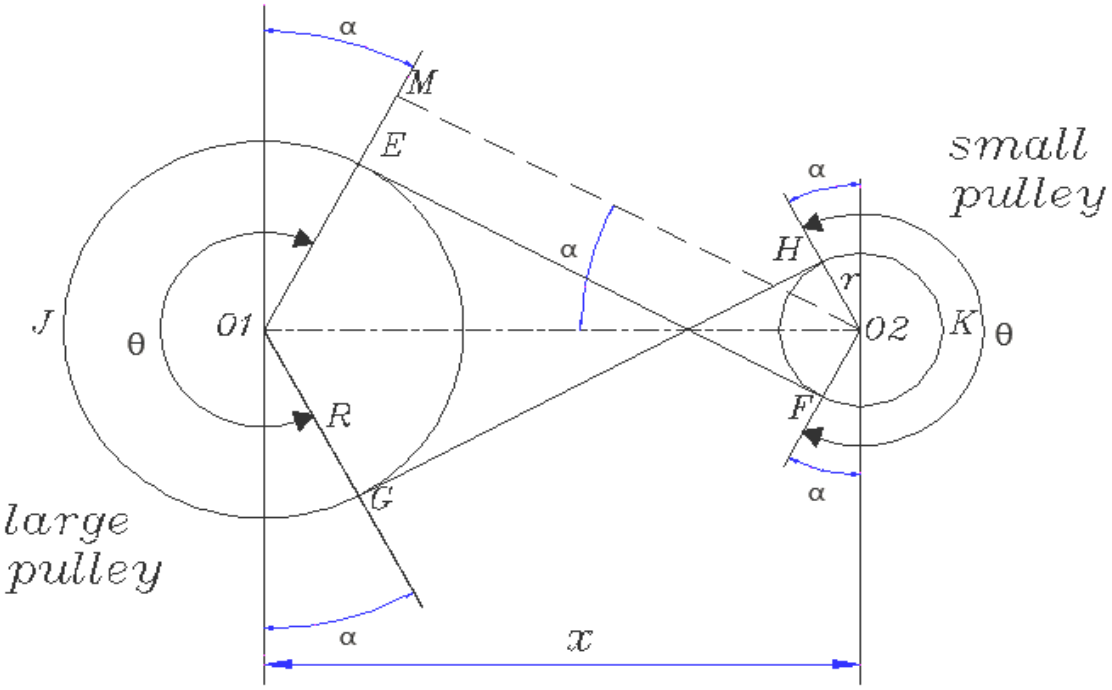
$$\boxed{L = \pi(R+r)+2x+\frac{(R-r)^2}{x}}$$

The angle of contact θ

$$\boxed{\theta_{rad} = \frac{\pi}{180}(180-2\alpha)}$$ *For small pulley*

$$\boxed{\alpha = \sin^{-1}\left(\frac{R-r}{x}\right)}$$

Ⓑ Length of cross belt drive :



By the same way

$$L = \pi(R + r) + 2x + \frac{(R + r)^2}{x}$$

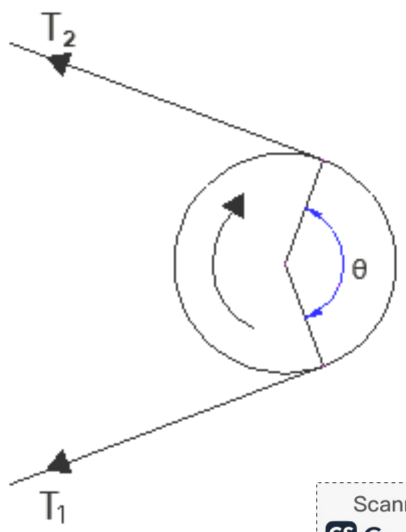
The angle of contact θ

$$\theta_{rad} = \frac{\pi}{180}(180 + 2\alpha)$$

For small pulley

$$\alpha = \sin^{-1}\left(\frac{R + r}{x}\right)$$

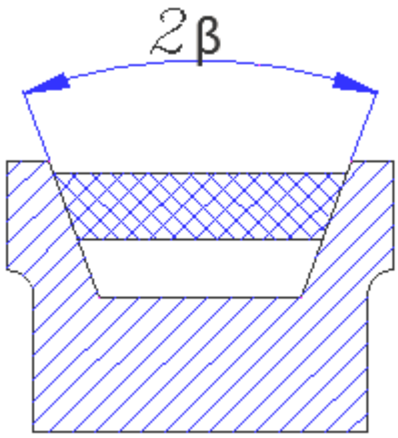
3-Ratio of driving tensions :



***T**1: tension of tight side. (N)*
***T**2: tension of slack side. (N)*
Ratio of tension forces depend on θ and μ

$$\frac{T_1}{T_2} = e^{\mu\theta}$$ For flat belt

$$\frac{T_1}{T_2} = e^{\left(\frac{\mu\theta}{\sin\beta}\right)}$$ For V- belt



Where θ: angle of contact (radian)
μ: coefficient of friction
2β: groove angle

4-Power transmitted by belt:

$$P = (T_1 - T_2)\cdot V$$ Watt

$$P = \frac{(T_1 - T_2)\cdot V}{750}$$ h.p

$$V = \frac{2\cdot\pi\cdot N_1}{60}\cdot R = \frac{2\cdot\pi\cdot N_2}{60}\cdot r$$

V : velocity of belt (m/s)

Example (1)

*Two pulleys with diameters **(120 mm)** and **(100 mm)**, centre distance **(300 mm).** Determine the length of belt in case of open and cross belt drive.*

Solution:

$R = 60mm \quad , \quad r = 50mm \quad , \quad x = 300mm$

In case of open belt drive

$$L = \pi(R + r) + 2x + \frac{(R - r)^2}{x}$$

$$L = \pi(60 + 50) + 2 \times 300 + \frac{(60 - 50)^2}{300} = 946mm$$

In case of cross belt drive

$$L = \pi(R + r) + 2x + \frac{(R + r)^2}{x}$$

$$L = \pi(60 + 50) + 2 \times 300 + \frac{(60 + 50)^2}{300} = 986mm$$

Example (2)

*Two pulleys with diameters **(450 mm)** and **(200 mm)**, centre distance **(1.95 m)**. if we use cross belt drive. Determine the length of flat belt, and the contact angle between belt and smaller pulley. Also determine the horse power. If the large pulley turns with speed **(200 rpm)** and the tension force in tight side is **(1000 N),** coefficient of friction **μ=0.25***

Solution:

$R = 0.225m \quad , \quad r = 0.1m \quad , \quad x = 1.95m$

$N_1 = 200.rpm \quad , \quad T_1 = 1000N \quad , \quad \mu = 0.25$

$$L = \pi(R + r) + 2x + \frac{(R + r)^2}{x}$$

$$L = \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975m$$

$$= 4975mm$$

$$\alpha = \sin^{-1}\left(\frac{R+r}{x}\right) = \sin^{-1}\left(\frac{0.225+0.1}{1.95}\right) = 9.6^o$$

$$\theta_{rad} = \frac{\pi}{180}(180+2\alpha) = 3.477\,rad$$

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \Longrightarrow \quad \frac{1000}{T_2} = e^{0.25\times 3.477}$$

$$T_2 = 419.3N$$

$$V = \frac{2\cdot\pi\cdot N_1}{60}\cdot R = \frac{2\times\pi\times 200}{60}\times 0.225 = 4.7m/s$$

$$P = \frac{(T_1-T_2)\cdot V}{750} = \frac{(1000-419.3)\times 4.7}{750} = 3.64h.p$$

Example (3)

The diameter of Two pulleys are (0.3 m) and (0.2 m), centre distance (1 m). if we use open belt drive. Determine

1- *The length of V- belt. ($2\beta=60^o$)*
2- *The contact angle between belt and smaller pulley.*
3- *The horse power. If the smaller pulley turns with speed (400rpm) and the tension force in tight side (T_1= 900N), coefficient of friction* ***μ=0.3***
4- *The speed of the larger pulley*

SOLUTION :

$R = 0.15m$, $r = 0.1m$, $x = 1m$

$N_2 = 400.rpm$, $T_1 = 900N$, $\mu = 0.3$

1-

$$L = \pi(R+r) + 2x + \frac{(R-r)^2}{x}$$

$$L = \pi(0.15+0.1) + 2\times 1 + \frac{(0.15-0.1)^2}{1} = 2.79m$$

$$= 2790mm$$

2-

$$\alpha = \sin^{-1}\left(\frac{R-r}{x}\right) = \sin^{-1}\left(\frac{0.15-0.1}{1}\right) = 2.87^o$$

$$\theta_{rad} = \frac{\pi}{180}(180-2\alpha) = 3.04 rad$$

3-

$$\frac{T_1}{T_2} = e^{\frac{\mu\theta}{\sin\beta}} \Longrightarrow \frac{900}{T_2} = e^{\frac{0.3\times3.04}{\sin 30}}$$

$$T_2 = 145.2N$$

$$V = \frac{2\cdot\pi\cdot N_2}{60}\cdot r = \frac{2\times\pi\times400}{60}\times0.1 = 4.2 m/s$$

$$P = \frac{(T_1 - T_2)\cdot V}{750} = \frac{(900-145.2)\times4.2}{750} = 4.23 h.p$$

4-

$$V = \frac{2\cdot\pi\cdot N_1}{60}\cdot R = \frac{2\cdot\pi\cdot N_2}{60}\cdot r$$

$$N_1\cdot R = N_2\cdot r$$

$$N_1\times0.15 = 400\times0.1$$
$$N_1 = 266.7 rpm$$

Clutch

Function: it is a machine part which translates or transmits motion from one part to another.

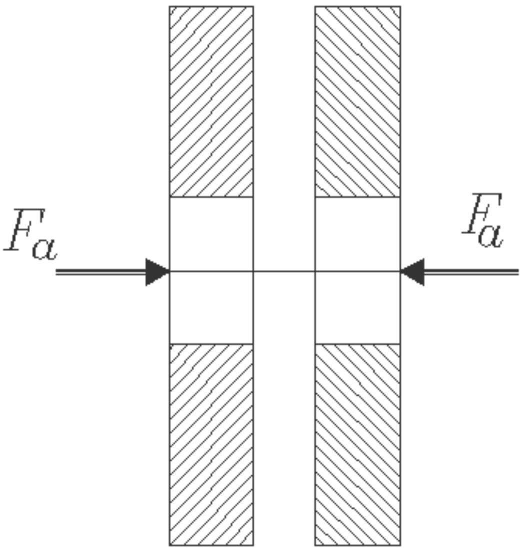
Types of clutches:

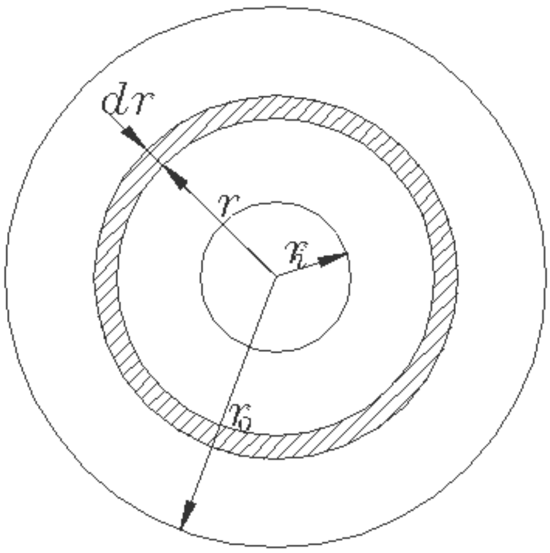
I) friction clutch: Divide into: → *Disc clutch* → *single disc*; → *Multiple discs*
→ *Cone clutch*

II) Hydraulic clutch
III) Electro –magnetic clutch
IV) Automatic clutch

I) Friction clutch

Consider two flat surfaces, maintained in contact by axial thrust (F_a)





Let
r_o *:outer radius of clutch (m)*
r_i*: inner radius of clutch (m)*
T_f *: torque transmitted by friction (N.m)*
P: axial pressure (N/m^2)
F_a*: axial force (thrush force) (N)*
μ*: coefficient of friction.*

The force acting on element $=dF_a$

$$dF_a = p\cdot(2\pi\cdot r\cdot dr)$$

$$dF_f = \mu\cdot dF_a$$

$$dT_f = dF_f\cdot r$$

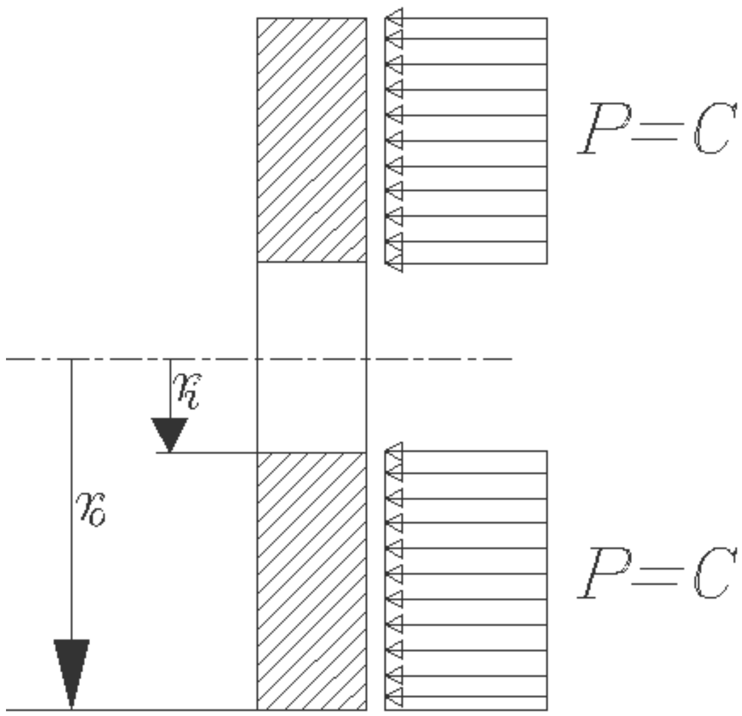
$$dT_f = p\cdot\mu\cdot 2\pi\cdot r^2\cdot dr$$

The design of friction clutch must be done on the base of

1- Uniform pressure: (new clutch)

$$P = constant = P_{max}$$

$$dF_a = p\cdot(2\pi\cdot r\cdot dr)$$



$$F_a = \int_{r_i}^{r_o} p\cdot(2\pi\cdot r\cdot dr) = p\cdot\left[2\pi\cdot\frac{r^2}{2}\right]_{r_i}^{r_o}$$

$$\boxed{F_a = \pi\cdot p_{\max}\cdot\left(r_o^2 - r_i^2\right)}$$

$$dT_f = p \cdot \mu \cdot 2\pi \cdot r^2 \cdot dr$$

$$T_f = p_{\max} \cdot \mu \int_{r_i}^{r_o} 2 \cdot \pi \cdot r^2 \cdot dr = p_{\max} \cdot \mu \cdot \left[\frac{2}{3} \cdot \pi \cdot r^3\right]_{r_i}^{r_o}$$

$$T_f = p_{\max} \cdot \mu \cdot \frac{2}{3} \cdot \pi \cdot \left(r_o^3 - r_i^3\right)$$

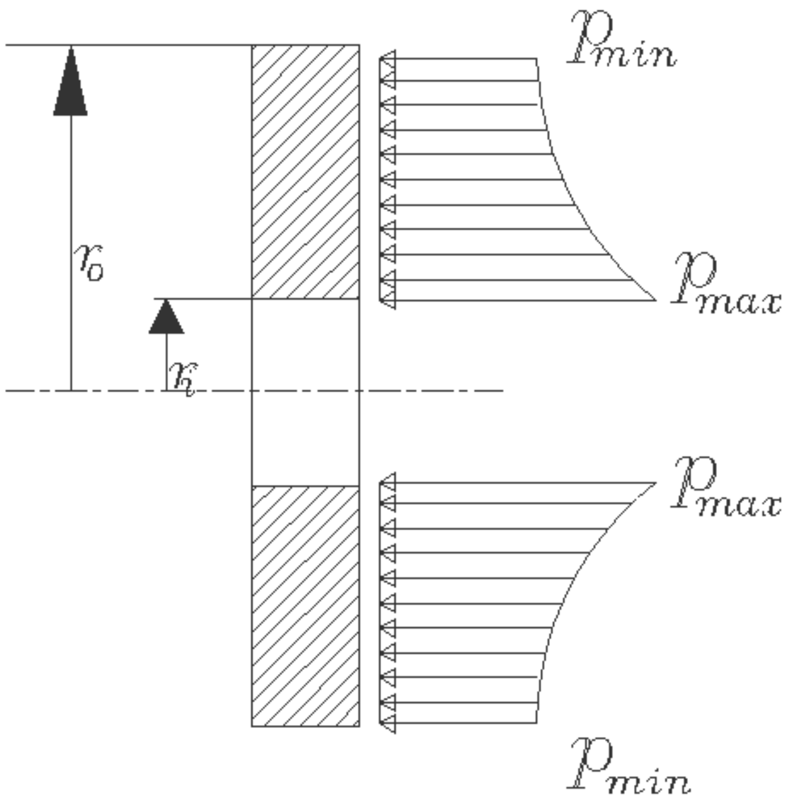
$$T_f = \frac{2}{3} \cdot \left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right) \cdot \mu \cdot F_a$$

2-Uniform wear: (old clutch)

$$p \cdot r = constsnt$$

$$p_{\max} \cdot r_i = C$$

$$p_{\min} \cdot r_o = C$$



$$dF_a = p \cdot (2\pi \cdot r \cdot dr)$$

$$F_a = 2\pi \int_{r_i}^{r_o} p \cdot r \cdot dr = 2 \cdot \pi \cdot C \int_{r_i}^{r_o} dr = 2 \cdot \pi \cdot C \cdot (r_o - r_i)$$

$$F_a = 2 \cdot \pi \cdot C \cdot (r_o - r_i)$$

$$dT_f = p \cdot \mu \cdot 2\pi \cdot r^2 \cdot dr$$

$$T_f = 2\pi \cdot \mu \int_{r_i}^{r_o} p \cdot r^2 \cdot dr = 2\pi \cdot \mu \int_{r_i}^{r_o} p \cdot r \cdot r \cdot dr$$

$$T_f = 2\pi \cdot \mu \cdot C \cdot \int_{r_i}^{r_o} r \cdot dr = 2\pi \cdot \mu \cdot C \cdot \left[\frac{r_o^2}{2} - \frac{r_i^2}{2}\right]$$

$$\boxed{T_f = \pi \cdot \mu \cdot C \cdot \left(r_o^2 - r_i^2\right)}$$

$$\boxed{T_f = \mu \cdot F_a \cdot \left(\frac{r_o + r_i}{2}\right)}$$

Consideration must be taken in design of friction clutch:

*1-for multi-disc clutch let **n** be the number of pairs of contact surfaces.*

$$T_f = n \cdot \mu \cdot F_a \cdot R_f$$

$$R_f = \frac{2}{3} \cdot \left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right) \qquad \text{For uniform pressure}$$

$$R_f = \frac{r_o + r_i}{2} \qquad \text{For uniform wear}$$

R_f : friction radius

2- if there are n_1: number of discs on the driver
n_2: number of discs on the driven
Then the number of pairs of contact surface

$$\boxed{n = n_1 + n_2 - 1}$$

3-Recommanded the ratio

$$0.6 < \frac{r_i}{r_o} < 0.8$$

4- Friction torque must be more than engine torque

$$T_f = \beta \cdot T_{engine}$$

β :engagement factor

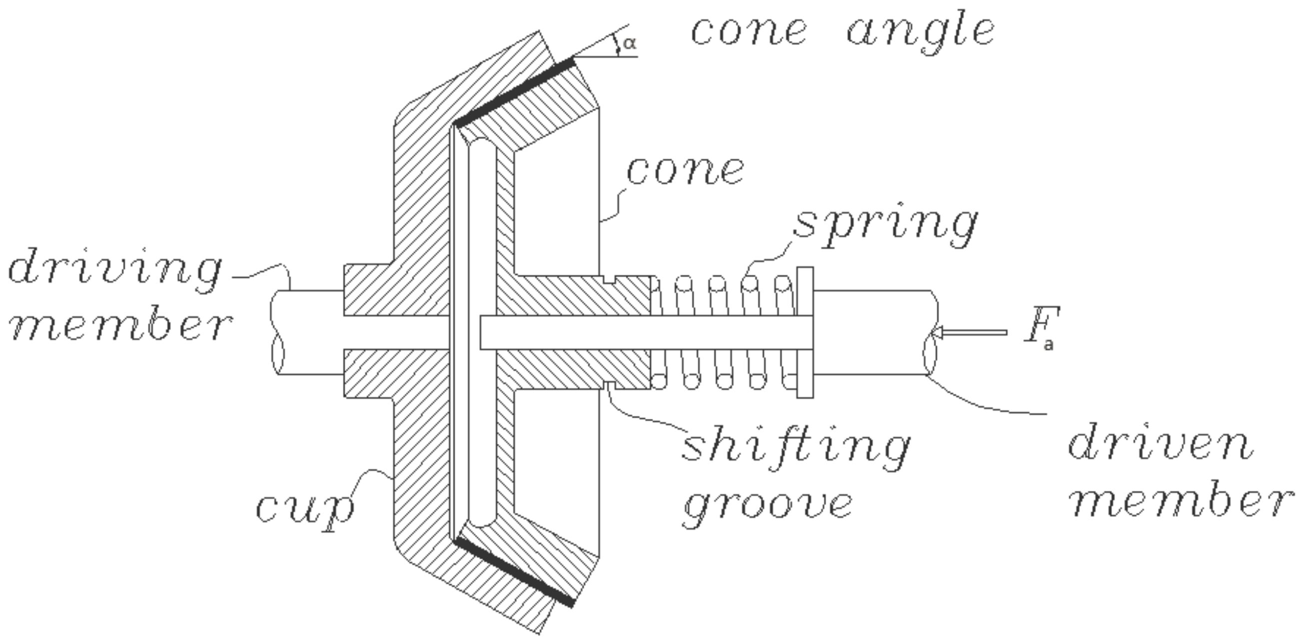
$\beta = 1.25 \rightarrow 1.5$ For machine

$\beta = 1.2 \rightarrow 1.5$ For car

$\beta = 2 \rightarrow 2.5$ For tractors

Cone Clutch :

It is used to transmit high torque because of large friction area. specially used with low peripheral speed.



Let

α :cone face angle

$$\alpha = 8^o \rightarrow 12.5^o$$

F_a: axial force (spring force) (N)

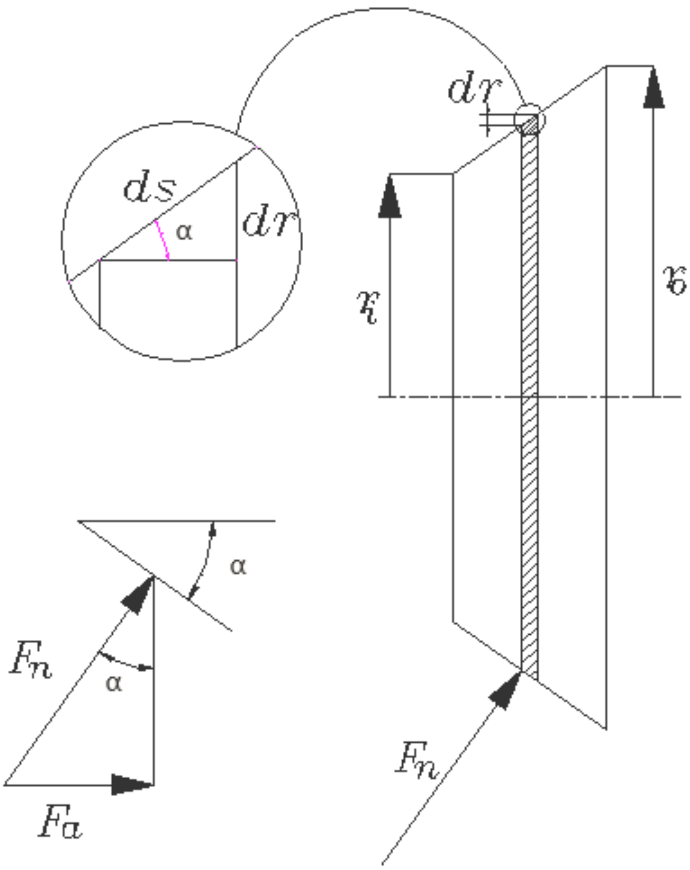
P_n: normal pressure (N/m^2)

F_n: normal Force (N)

$$\sin\alpha = \frac{F_a}{F_n}$$

$$F_a = F_n \cdot \sin\alpha$$

Element of area $$dA = 2\pi \cdot r \cdot ds = 2\pi \cdot r \cdot \frac{dr}{\sin\alpha}$$



$$dF_n = p_n \cdot \left(2\pi \cdot r \cdot \frac{dr}{\sin\alpha}\right)$$

$$dF_a = p_n \cdot \left(2\pi \cdot r \cdot \frac{dr}{\sin\alpha}\right) \cdot \sin\alpha$$

$$dF_a = 2\pi \cdot p_n \cdot r \cdot dr$$

$$dT_f = dF_{n} \cdot \mu \cdot r = 2\pi \cdot \mu \cdot p_n \cdot r^2 \cdot \frac{dr}{\sin\alpha}$$

1-uniform pressure $p_n = C$

$$dF_a = 2\pi \cdot p_n \cdot r \cdot dr$$

$$F_a = 2\pi \cdot p_n \int_{r_i}^{r_o} r \cdot dr = 2\pi \cdot p_n \cdot \left[\frac{r^2}{2} \right]_{r_i}^{r_o}$$

$$F_a = \pi \cdot p_n \cdot \left(r_o^2 - r_i^2 \right)$$

$$dT_f = 2\pi \cdot \mu \cdot p_n \cdot r^2 \cdot \frac{dr}{\sin\alpha}$$

$$T_f = 2\pi \cdot \mu \cdot p_n \cdot \frac{1}{\sin\alpha} \int_{r_i}^{r_o} r^2 \cdot dr = 2\pi \cdot \mu \cdot p_n \cdot \frac{1}{\sin\alpha} \left[\frac{r^3}{3} \right]_{r_i}^{r_o}$$

$$T_f = \frac{\pi \cdot p_n \cdot \mu}{\sin\alpha} \cdot \frac{2}{3} \cdot \left(r_o^3 - r_i^3 \right)$$

$$T_f = \frac{2}{3} \cdot \left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \right) \cdot \frac{\mu \cdot F_a}{\sin\alpha}$$

2-uniform wear $p_n \cdot r = C$

$$dF_a = 2\pi \cdot p_n \cdot r \cdot dr$$

$$F_a = 2\pi \int_{r_i}^{r_o} p_n \cdot r \cdot dr = 2 \cdot \pi \cdot C \int_{r_i}^{r_o} dr = 2 \cdot \pi \cdot C \cdot \left(r_o - r_i \right)$$

$$F_a = 2 \cdot \pi \cdot C \cdot \left(r_o - r_i \right)$$

$$dT_f = 2\pi \cdot \mu \cdot p_n \cdot r^2 \cdot \frac{dr}{\sin\alpha}$$

$$T_f = 2\pi \cdot \mu \cdot \frac{p_n \cdot r}{\sin\alpha} \int_{r_i}^{r_o} r \cdot dr$$

$$T_f = \frac{2\pi \cdot \mu \cdot C}{\sin\alpha} \cdot \left[\frac{r_o^2}{2} - \frac{r_i^2}{2} \right]$$

$$T_f = \frac{\pi \cdot \mu \cdot C}{\sin\alpha} \cdot \left(r_o^2 - r_i^2 \right)$$

$$T_f = \frac{\mu \cdot F_a}{\sin\alpha} \cdot \left(\frac{r_o + r_i}{2}\right)$$

Example (1)

*A multi-disc clutch employ **3** steel and **2** bronze disc having outer diameter **(30 cm)** and inner diameter **(20 cm)**. Find the axial force and the power transmitted if the normal pressure is **(P = 1.3 kg /cm^2)** and **(N=750 rpm)**.take uniform wear **(μ =0.22)***

Solution :

$r_o = 15cm = 0.15m \quad , \quad r_i = 10cm = 0.1m$

$p_{max} = 1.3\dfrac{kg}{cm^2} = 13\dfrac{N}{cm^2} = 13\dfrac{N}{\dfrac{m^2}{10^4}} = 13\times10^4\dfrac{N}{m^2} \quad , \quad N = 750rpm$

Uniform wear , $\mu = 0.22$

$$F_a = 2\cdot\pi\cdot C\cdot(r_o - r_i)$$
$$= 2\cdot\pi\cdot(p_{max}\cdot r_i)\cdot(r_o - r_i)$$
$$F_a = 2\times\pi\times(13\times10^4\times0.1)\times(0.15-0.1) = 4084N$$
$$n = n_1 + n_2 - 1$$
$$n = 3 + 2 - 1 = 4$$

$T_f = n\cdot\mu\cdot F_a\cdot R_f \qquad \because R_f = \dfrac{r_o + r_i}{2}$

$$T_f = 4\times0.22\times4084\times\left(\frac{0.15+0.1}{2}\right) = 449.24N\cdot m$$

Power transmitted $= P = T_f \cdot \omega$

وزارة التعليم العالي والبحث العلمي جامعة
الفرات الاوسط التقنية
المعهد التقني/كربلاء
القسم/ التقنيات الميكانيكية
الفرع/ الانتاج

تقنية أجزاء المكائن

السنة الثانية

الجزء الثاني

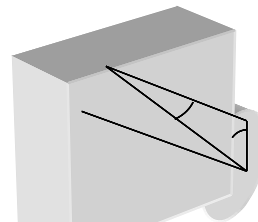
Torsional stress

م. م. انتصار رشيد صالح الخرسان

ماجستير هندسة ميكانيكية

Torsional stress:

It is the action of two equal and opposite couples acting in parallel planes, and the stress is known as torsional Shear, which varies from zero at the centre to the maximum value at the outer fibre.

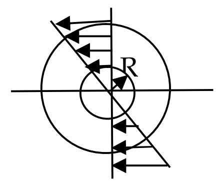


General equation for torsion:

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

Shear stress for Solid rcalar cross section is btained by the equation

$$\tau = \frac{T \times R}{J} \qquad J = \frac{\pi \times d^4}{32}$$



T:applied Torqe (N.M)

J:polar moment of area (m^4)

torsional Shear Stress(N/M^2)τ:

r:distanace from center to the point which the Stress may be calculated.(m)

R:Radius of cross section area(m)

G:modulus of rigidity(N/M^2)

L:Lenth of bar (m)

Ө:angle of twist.

EX:1

Hollow Shaft(50mm)outer diameter,(30mm)insider diameter is subjected to torque of (1600N.M) and has angle of twist(0.4^o)modulus of rigidity($86G/M^2$). Determine the length of Shaft If the Shaft turn

with(2000rpm).Determine the maximum transmitted power .take the allowable torsion Shear Stress (65MN/M^2)?

SOLUTION:

D_o=50mm=0.05m ,d_i=30mm=0.03m ,T=1600N.M

$\Theta=0.4=0.4\times\frac{\pi}{180}=6.98\times10^{-3}$rad,$\tau=65MN/M^2=65\times10^6$N/m

G=86GM/M^2=86×10^{9}N/M^2 , N=2000rpm

$J=\frac{\pi}{32}(d_o^4-d_i^4)=\frac{\pi}{32}[(0.05)^4-(0.03)^4]$

$J=5.34\times10^{-7}m^4$

$\frac{T}{J}=\frac{G\Theta}{L}$ $L=\frac{G\Theta J}{T}$ $L=\frac{86\times10^9\times6.98\times10^{-3}\times5.34\times10^{-7}}{1600}=0.2m$

2-

$$\tau=\frac{T.R}{J} \qquad T=\frac{\tau.J}{R}$$

$$T=\frac{65\times10^6\times5.34\times10^{-7}}{\frac{0.05}{2}}=1388.4N.M$$

$$\omega=\frac{2\pi N}{60}=\frac{2\pi2000}{60}=209.4rad/s$$

Power $P=T\times\omega$

=1388.4×*209.4*

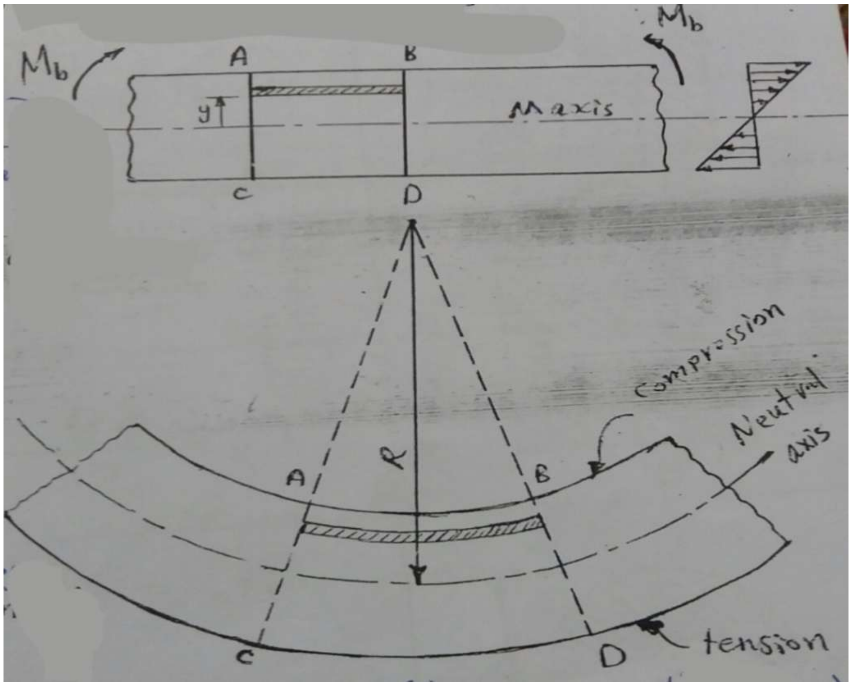
=290731 watt

***P*=290.731 k**

Bending Stress:

Bending Stress produces when the part is bent as shown the outer fibres are under tension .The inner fibres under compression and the intermediate layer with no stress .

$$\frac{M_b}{I}=\frac{\sigma_b}{y}=\frac{E}{R}$$



From general equation for Bending

$\sigma_{b} = \frac{M_{b} \times y}{I}$

M_b: bending moment(N.M)

y: distance from neutral axis to layer which to be calculate the stress(m)

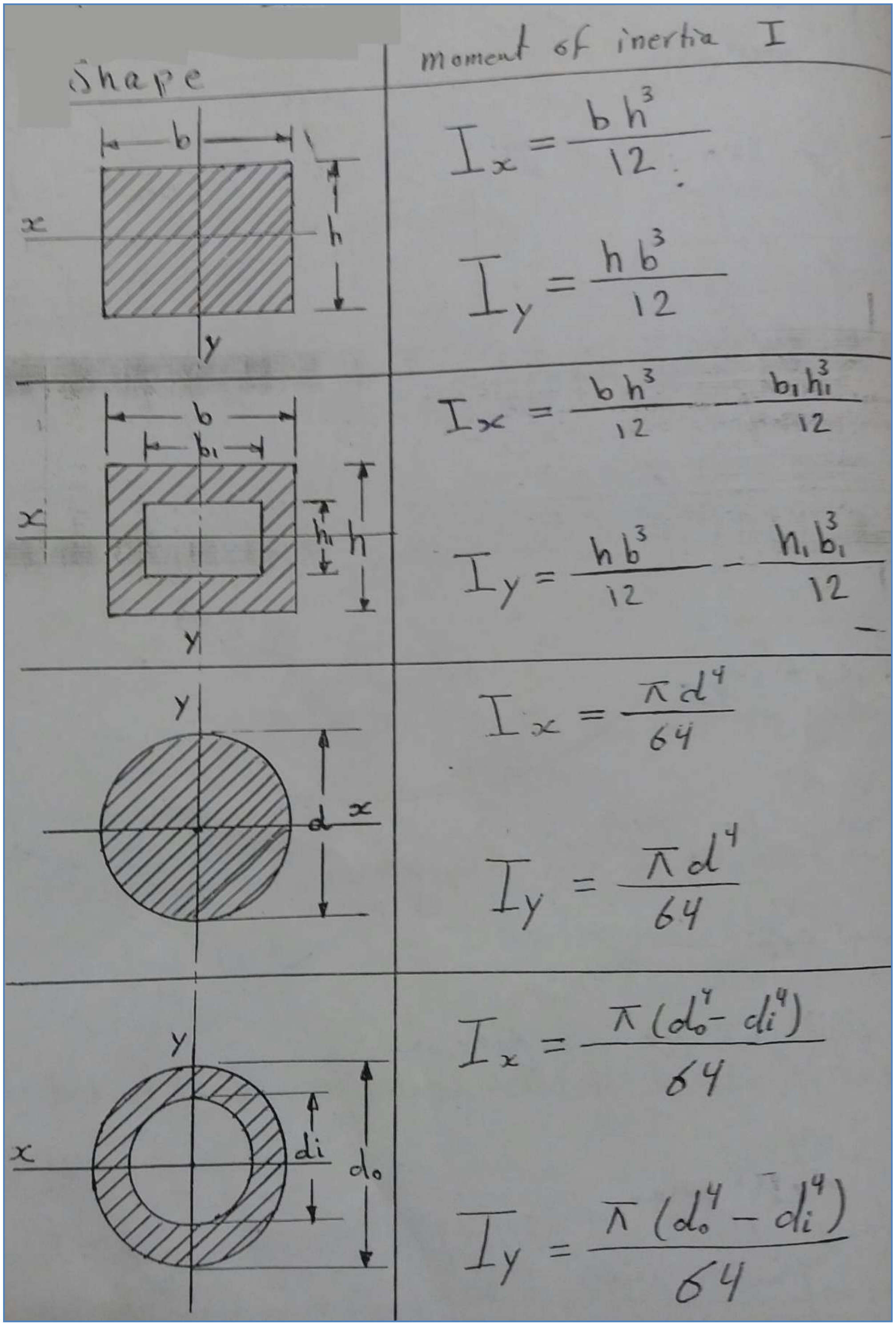
σ_b : bending stress (N/M^2)

I: Moment of inertia of cross section area about neutral axis(m^4)

E: modulus of elasticity (N/M^2)

R: radius of curvature (m)

<u>Moment of Inertia:</u>



Shape	moment of inertia I
b, h, x, y	$I_x = \frac{b h^3}{12}$ $I_y = \frac{h b^3}{12}$
b, b_1, h_1, h, x, y	$I_x = \frac{b h^3}{12} - \frac{b_1 h_1^3}{12}$ $I_y = \frac{h b^3}{12} - \frac{h_1 b_1^3}{12}$
y, d, x	$I_x = \frac{\pi d^4}{64}$ $I_y = \frac{\pi d^4}{64}$
y, x, di, d_o	$I_x = \frac{\pi (d_o^4 - d_i^4)}{64}$ $I_y = \frac{\pi (d_o^4 - d_i^4)}{64}$

Ex1

Attached to bar shown with circutar cross section .If the bending stress of bar material (65MN/m2) Determine the diameter of the bar .

<u>Solution</u>

∑MB = +

RA*57-30*45-40*10=0

RA*57=30*45+40*10

RA=30.7kN

∑Fy = 0 ↑ +

RA-30-40+RB =0

30.7-30-40+RB = 0

RB =39.3KN

Bending moment diagram B.M.D

MA =RA*0 =0

MC =RA*12-30*0 =368.4 KN.cm

MD = RA*(12+35) -30*35-40*0 = 392.9 KN.cm

MB =RA*(12+35+10)-30*(35+10)-40*10 = 0

Or

MA = 0

Mc = 30.7*12 = 368.4 KN.cm

MD =368.4+0.7*35 = 392.9KN.cm

MB =392.9-39.3 *10 = 0

The max. Bending moment is (392.9 KN.cm)

at point D

$$\sigma b = \frac{Mb * y}{I}$$

Where Mb =392.9KN.cm

$$=392.9*10^3 \text{ N.cm}$$

$$=\frac{392.9*1000}{100} \text{ N.m}$$

Mb =3929 N.m

$y = \frac{d}{2}$ **for circular section**

$$I = \frac{\pi}{64} d^4$$

$$\sigma b = \frac{Mb*y}{I}$$

$$65*10^6 = \frac{3929*\frac{d}{2}}{\frac{\pi}{64}}$$

$$d^3 = \frac{3929*\frac{1}{2}}{\frac{\pi}{64}}$$

=0.000616

d = 0.0851 m =85.1 mm

Shafts

Shaft is a rotating member which transmits power from one point to another points .

Material

For shaft with diameters

(a) d =(7.6 → 8.9 cm) from cold rolled carbon stell

(b) d > 8.9cm from hot rolled carbon steel

Design of shaft :-

The shaft are designed on the base of bending or torsion or (bending + torion) stresses as follow

Shaft subjected to torsion

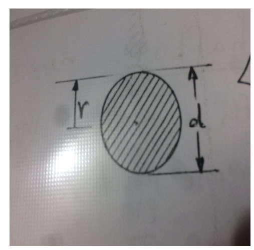
By using Torsion only

$$\tau = \frac{T.r}{J}$$ **T:Torque** **(N.m)**

τ: shear stress ($\frac{N}{M2}$)

J : polar moment of Inertin (m^4)

For solid shaft

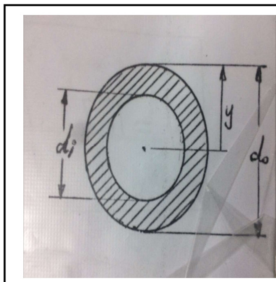


$$J=\frac{\pi}{32}d^4 \quad , r = \frac{d}{2}$$

$$\tau= \frac{T.r}{J} \qquad \text{Or } d=1.72 \sqrt[3]{\frac{T}{\tau}}$$

FOR HOLLOW SHAFT

$$J=\frac{\pi}{32}(d^4{}_o - d^4{}_i) \quad , r = \frac{d.}{2}$$



$$\tau = \frac{T.r}{J} = \frac{T.\frac{d.}{2}}{\frac{\pi}{\frac{\pi}{32}(d4o - d4i)}}$$

$$\tau = \frac{16 \quad d.}{\pi(d4o - d4i)} = \frac{16T\ d.}{\pi d4o\ (1-\frac{di}{d0})} = \frac{16T}{\pi d3o\ (1-\frac{di}{d0})}$$

$$\tau = \frac{16T}{\pi d3o\,(1-\frac{di}{d0})} \quad OR\ d. = 1.72\sqrt[3]{\frac{T}{\tau(1-\left(\frac{di}{do}\right)^4}}$$

shaft subjected to bending only :-

by using bending eqyation

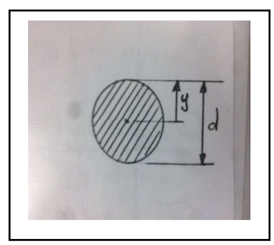
$\sigma_b = \frac{Mb\ .\ y}{I}$ $\sigma_{b:}$ bending stress (pa)

M_b: max bending moment (N.m)

I : moment of interia (m^2)

$M_b = \sqrt{Mr^2 + Mh^2}$

For solid shaft



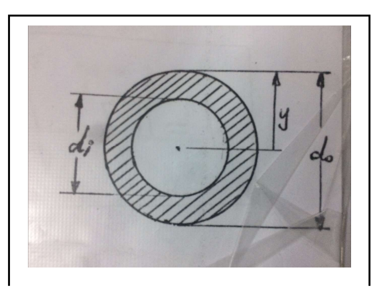
Y =d / 2 ,,,, $J = \frac{\pi}{32} d^4$

$$\sigma_b = \frac{Mb\ .\ y}{I} = \frac{Mb\ .\ \frac{d}{2}}{\pi \quad d^3} = \frac{32Mb}{\pi d^3}$$

$$\sigma_b = \frac{32Mb}{\pi d^3} \quad or \quad d = 2.17\sqrt[3]{\frac{Mb}{\sigma b}}$$

for hollow shaft

$y = \frac{do}{2}$, $I = \frac{\pi}{64}(d^4{}_o - d^4{}_i)$



$$\sigma b = \frac{Mb*y}{I} = \frac{Mb\frac{do}{2}}{\frac{\pi}{64}(do^4 - di^4)} = \frac{32Mb*do}{\frac{\pi}{64}(do^4 - di^4)}$$

or $$do = 1.72\sqrt[3]{\frac{Mb}{\left(1-\left(\frac{di}{do}\right)^4\right)\sigma b}}$$ $$\sigma b = \frac{32Mb}{\pi d^3\left(1-\left(\frac{di}{do}\right)^4\right)}$$

III Shaft subjected to(Bending+torsion)stress : $\tau max = \frac{1}{2}\sqrt{\sigma b^2 + 4T^2}$

For solid shaft

$$\tau max = \frac{1}{2}\sqrt{\left(\frac{32\,Mb}{\pi d^3}\right)^2 + 4\left(\frac{16\,T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\sqrt{Mb^2 + T^2}$$

$$\tau max = \frac{16\,Te}{\pi b^3} \qquad Te = \sqrt{Mb^2 + T^2}$$

$$d = 1.72\sqrt[3]{\frac{Te}{\tau max}}$$

Te : equivalent Torq

N.m

For Hollow shaft

$$\tau max = \frac{1}{2}\sqrt{\left(\frac{32\,Mb}{\pi do^3\left(1-\left(\frac{di}{do}\right)^4\right)}\right) + 4\left(\frac{16\,T}{\pi do^3\left(1-\left(\frac{di}{do}\right)^4\right)}\right)}$$

$$\tau max = \frac{16}{\pi do^3\left(i-\left(\frac{di}{do}\right)^4\right)}\sqrt{Mb + T^2}$$

$$\tau max = \frac{16\,Te}{\pi do^3\left(1-\left(\frac{di}{do}\right)^4\right)} \qquad Te = \sqrt{Mo^2 + T^2}$$

$$do = 1.72\sqrt[3]{\frac{Te}{\left(1-\left(\frac{di}{do}\right)^4\right)\tau max}}$$

Ex1 - For the shaft (circular solid as shown determine the diameter of shaft if the allowable stress for material ((σ_b=125 Mpa))

$M_{B} = 0$

$R_a * 75 – 5 * 50 – 50 * 25 = 0$

$R_a = 5KN$

$F_y = 0$

$R_A – 5 – 5 + R_B = 0$

$R_B = 5KN$

$M_A = 5KN * 25 CM$

$M_C = 125KN .CM$

$M_D = 125 KN.CM$

$M_B = 125KN.CM – 5 * 25$

$M_B = 0$

THE MAX BENDING MOMENT

M_B = 124KN.CM = 124 * 10^3 N . CM

$M_B = \frac{125000}{100} N.M = 1250$ PA

$M_B = \frac{32Mb}{\pi d^3} \quad 125 * 10^6 = \frac{32*1250}{\pi d^3}$

d= 0.0467 m

Ex2

Ex2 - For the solid shaft as shown: determine the diameter of shaft if the shaft transmitted power of 40kw at 300 rpm , take the allowable stress material (τ = 42Mpa)

Sol

Bending torsion

$M_B = 0$

$R_A = * 60 – 3* 40 – 4* 20 = 0$

$R_{A} = 3.33KN$

$F_Y = 0$

R_A-3-4+R_B=0

R_B=3.76

M_A= 0

M_C= 3.33* 20 = 6.66KN.CM

M_D= 6.66+ 0.33*20 = 73.2KNCM

M_B= 73.2 – 3.67*20

$M_{B\ =0}$

M_b= 73.2 * 1000 ÷ 100= 732

P= T*w

$4000 = T* \frac{2\pi*300}{60}$,

T = 1273.2 N. M

$T_e = = \sqrt{Mb^2 + T^2} \qquad \sqrt{(732)^2 + (1273.3)^2}$

T_e = 1468.6 N,m

$\tau = \frac{16Te}{\pi d^3}$

d= 0.0563m = 56.3 mm

or

$d = 1.72 \sqrt[3]{\frac{Te}{\tau max}} = 0.0562 \text{ m} = 56.2 \text{ mm}$

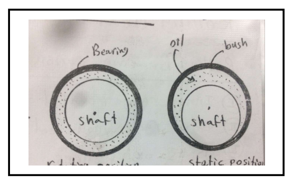
bearing

bearing it a machine part used to reduce or prevent friction between two parts .

Types of bearing on the base of contact

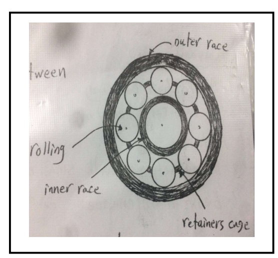
1 – sliding bearing

There is sliding contact between the surface of the supported member and turning member



2- rolling bearing

There is rolling contact between the two surfaces



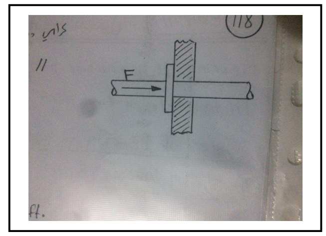
Tybe of bearing on the base of load

1- radial bearing

in this tybe the load act to the axis of rotation

2- thrust bearing

in the tybe the load act   to the axis of rotation.



Rolling Bearing consist

1- inner race mounted on shaft
2- inner race mounted on housing
3- balls or rollers
4- retainers

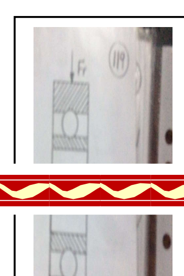
advantage

1- antifriction bearing
2- it can carry heavy small large site
3- easy maintenance

disadvantage

1- High cost
   Extreme care in manufacture

Tybe of ball bearing

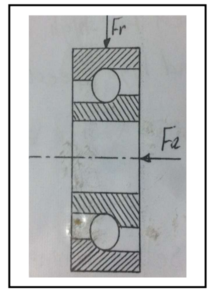


1- Deep groove ball bearing

Used for radial load and axial load at high speed

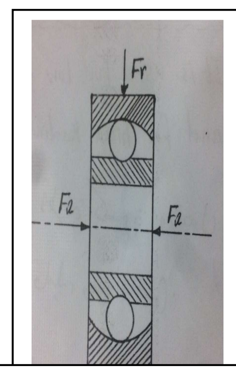
2- angular contact ball bearing

Used for axial load in one direction



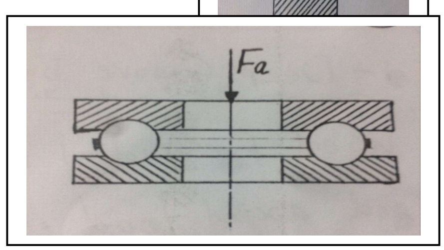
3. self aligning ball bearing

It used for shaft inclined by 2 – 3 and may be externally or interally self aligning



4-Thrust ball bearing

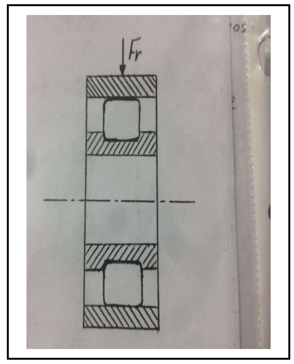
It used for axial load only



Types for rolles bearing

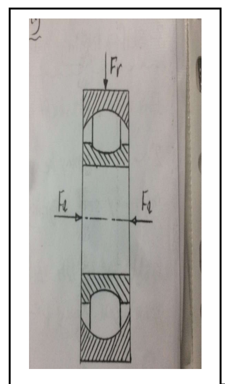
1- Cylindrical roller bearing

Used from high racliad load only and high speed



2- Self – aligning roller bearing

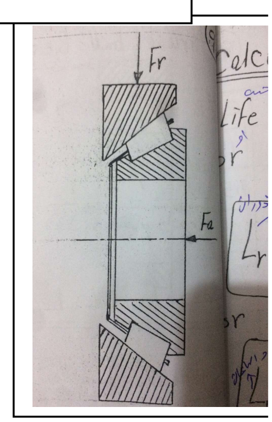
It is used for low axial load and for high radial load



3-Taper Roller Bearing .1

Used for high axral lend and Radial load

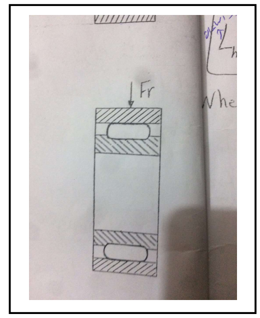
يتحمل قوه محوريه كبيره وباتجاه واحد فقط مع محصلة القوه القطرية



4-Needle Roller Bearing

Used for high load

يتحمل قوه قطرية فقط



<u>Calculation of service (fatigue) life of bearing</u>

Life of bearing is calculated in million revolution or hours by equation:

$L_r = (Fd/Fa)^p * 10^6$ revolution

$L_h = (Fd/Fa)^p * 10^6/60n$ hours

Where: Fd : Basic dynamic capacity (N)

Fe : equivalent bearing load (N)

P : exponent

P=3 for ball bearing

P=10/3 for roller Bearing

n : r.p.m

equivalent bearing load is combined of axial and radial bearing load and account by the equation

C1: shock and impact factor which depend on the type of load

V: race rotation factor

V=1 when inner race turn

V=1 .2 when outer race turn

X: radial factor

Y: axial factor

Fr: radial load

Fa: axial load

Ex1//Determine the number and dimension of single deep groove ball bearing if the radial load (Fr=400N) axial load (Fa=500N) speed (n=1600rpm) for five years life work ten hours

per   with stead (constant) load and inner race turn   take .56   Y=1 X=0

<u>Solution :</u>

Single row deep groove ball

Fr =400N   Fa=500N   n=1600rpm   X=0. 56   Y=1

Lh=5*300*10=15000 hours   Bearing life

Lr=Lh*n*60 = 15000*1600*60

Lr =$1.44*10^{9}$ rev

Steady (constant) ⟶ C1=1

Inner race turn ⟶ V=1

Equivalent load

Fe=C1*(V*X*Fr+Y*Fa)

Fe=1*(1*0.56*4000+1*5000)

Fe=7240N

and for dynamic load

$Lr=(Fd/Fe)^{P} * 10^{6}$ where P=3 for ball bearing

$1.44*10^{9}=(Fd/7240)^{3}*10^{6}$

$(1.44*10^{9}/10^{6})^{1/3}=Fd/7240$

Fd=81757N

From table we select the bearing number and dimensions according to Fd

Ex2// Single row angular contact ball bearing with number (NQ50)is used for horizontal compressor and carry radial load (Fr=2500N) and axial load (Fa=1500) Determine the life of bearing and assume the load with light shocks take V=1 Fd=53000N X=1 Y=0

<u>Solution:</u>

Fr=2500N Fa=1500N

Fd=53000N X=1 Y=0

light shocks ⟶ C1=1 .5 V=1

Fe=C1*(V*X*Fr+Y*Fa)

Fe=1.5*(1*1*2500+0*1500)

Fe= 3750N

for ball bearing P=3

$Lr =(Fd/Fa)^{p}*10^{6}$

$Lr = (53000/3750)^{3} *10^{6}$

$Lr = 2.82*10^{9}$ rev

Ex3// Design double row self aligning ball bearing with radial load ($Fr=7000N$) axial load ($Fa=2100N$) bearing life ($Lr=160*10^6 rev$) speed ($n=300rpm$) and with constant load

Take $C1=1$ $V=1$ $X=0.56$ $Y=3.5$

Solution:

double row self- aligning bell bearing

$Fr=7000N$ $Fa=2100N$ $Lr=160*10^6 rev$ $n=300rpm$

$C1=1$ $V=1$ $X=0.65$ $Y=3.5$

$Fe=C1(V*X*Fr+y*Fa)$

$Fe=1*(1*0.65*7000+3.5*2100)$

$Fe=11900N$

For ball bearing $P=3$

$Lr=(Fd/Fa)^p 10^6$

$(160*10^6/10^6)^{1/3}=Fd/11900$

$Fd=6460N$

From table we select the dimension according to Fd

Gears:

Gears are used to transmit high boner with high accuracy

Spur gear terminology:

Addendum Circle

It is the circle drawn through the top of the teeth

Pitch Circle:

It is an imaginary circle at which from the tooth profile and at which the tooth has maximum thickness the size of the gear is specified by the pitch circle diameter

Base Circle:

It is the circle at which produce the involute curve and appear only in involute gear system

$$Db = d\ COS\varphi$$

Addendum(add):

It is the radial distance of an from pitch circle to the top of the tooth

Dedndum (add):

It is the radial distance of a tooth from pitch circle to the bottom of the tooth

Dedendum Circle (root circle):

It is the circle drawn through the bottom of the teeth**Working depth:** it is the redial distance from addendum circle to the clearance circle .
or the sum of the addendum of two mating teeth.
Pressure angle ($\emptyset$) :it is the angle between the common normal to two gears teeth at the point of contact and the common tangent at the pitch point .
Circular Pitch (P_c) : It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth .

$P_c = \frac{\pi d}{T}$ **d** : pitch circle diameter

T: Number of teeth .

$P_{c1} = P_{c2}$

$= \frac{\pi d2}{T2}$ $\frac{d1}{d2} = \frac{T1}{T2} \frac{\pi d1}{T1}$

Diametric Pitch (pd) :

it is the ratio between the number of the teeth and the pitch circle diameter in (mm)

$Pd = \frac{T}{d}$

Module (m): it is the ratio between the pitch circle diameter in "mm" and the nmber of teeth.

$M = \frac{d}{T}$

Clearance : It is the redial distance from the top of the tooth to the bottom of the other tooth in a mesh gear .

Central Distance: it is the distance between the center of the gear in mesh .

$C = \frac{d1+d2}{2}$

Tooth Thickness (t): It is the width of the tooth means along the pitch circle .

Top land : it is the surface of the top of the tooth .

Face of the tooth : it is the surface of the tooth above the pitch circle.

Flanck of the tooth: it is the surface of the tooth below the pitch circle .

Standard Values :

add. = 1 m

ded = 1.157 m

working depth = 2m

whole depth = 2.157 m

Clearance = 0.157 m

Pressure angle Ø = 20°

Gear Force :

Spur gear force components are :

F_t: tangential force (N)

F_r : redial force (N)

F_n : Normal force or resultant gear force.

$F_t = F_n . \cos Ø$

$F_r = F_n . \cos Ø = F_t . \tan Ø$

Design of gear on the base of strength of gear teeth (Lewis Equation):

In order to determine the stresses acting on the tooth, we assume that the tooth is as a beam of uniform strength
and fixed from one end , then the weakest section of the tooth
is at section A.A. by Assuming that the total load is carried by one tooth and by negleding friction, the tangential force cause
bending stress as follow :

$$\sigma_b = \frac{M\ .Y}{I} = \frac{M\ .C}{I} = \frac{Fe\ .\ H\ .\ \frac{T}{2}}{\frac{bt^3}{12}} = \frac{6\,Ft\ .h}{bt^2}$$

$$F_t = \sigma_{b)all} * b * \frac{t^2}{6h} \qquad \text{maltioly by } \frac{Pc}{Pc}$$

$$F_t = \sigma_{b)all} * b * \frac{t^2}{6h\ .\ Pc} \quad Pc$$

Assume the ratio

$F_t = 6b * b * P_C * \mathbf{y}$ Lewis equation

F_t: Tangential Force (N)

$\sigma_{b)all}$:allowable bending stress (pa)

P_c : Circular pitch (m)
b: Width of tooth (m)
y : Lewis Factor , which depends on the number of teeth ,and is a function of tooth shape.

Ø	14 1/2°	20° full depth involute	20° stub involute
Lewis factor	0.154-0684/T	0.154-0.912/T	0.175-0.84/T

the allowable bendi ng stress may be calculated as follows :

$6_b = 6_s * K$

----

where : 6_s : static stress (N/M^2)

K : velocity factor which depends on the value of pitch line velocity (V) is shown as in the table .

V	10 m/s	20 m/s	For non metallic gear
K	$3/3+V$	$6/6+V$	$0.75/1+V$ +0.25

$V = \frac{\pi d n}{60}$

$P = Ft * V$

Notes : Lewis equation is used to calculate the gears with (b).

EX/ A spur gear in mesh with pressure angle ($\emptyset = 14.5°$)Full depth involute profile of tooth, model (m 10), pitch diameter of pinion (d_p = 160 mm). Velocity ratio = R_V= 2/3. Calculate?

1. Number of teeth for each gear
2. Addendum.
3. Whole depth.
4. Clearance.
5. Outside diameter
6. Root diameter.
7. Duodenum
8. Base cycle diameter.

Solution :

$\emptyset =$ 14.5° , m= 10 , dp = 160mm R_V= 2/3

$$R_V = \frac{dp}{dg} = \frac{Tp}{Tg} = \frac{Ng}{NP} = \frac{2}{3}$$

$$R_V = \frac{2}{3} = \frac{dp}{dg} \text{ -------}\rightarrow \frac{2}{3} = \frac{160}{dg} \text{ --------}\rightarrow \text{dg= 240 m}$$

$$M = \frac{dp}{Tp} \text{ -------}\rightarrow \text{Tp} = \frac{dp}{M} = \frac{160}{10} = \text{16 teeth.}$$

$$M = \frac{dg}{Tg} \text{ ---------}\rightarrow \text{Tg} = \frac{dg}{M} = \text{240/ 10 ….. 24 teeth.}$$

Add . = m = 10 mm

whole depth = 2.157m = 21. 57 m

clearance = 0.157m = 1.57 mm

$D_{op} = d_p$ +2add. =180mm

$D_{og} = d_g$ +2add. = 260mm

Root diameter = Do – 2*whole depth

d_{rp}= 180 – 2*21.57 =136.86mm

d_{rg} =260 – 2*21.57 =216.86mm

ded. = 1.157m = 11.57 mm

base circle diameter

$D_{bp} = d_p * \cos\emptyset$=154.9mm

$D_{by} = d_y * \cos\emptyset$=232.36 mm

Ex/ 2. A bronze super gear pinion with static stress (6s =83 …..) Number of teeth (T_p= 16)and turn with speed (N= 600) drives a cast steel gear with a static stress (6=103M_N/m^2). If the pressure angle ($\emptyset =$ $20°$) $full$ depth involute profile of tooth , module (m=8) face width (b= 90mm) and velocity ratio (R_v= ¼)

Determine the power transmitted on the base of strength .

Solution :

6 = 83MN/m^2 T_p = 16 , N_p= 600 rpm rv= ¼

6 = 103M_N/m^2 $\emptyset = 20°$, M = 8 , b = 90mm

the design must be done on the base of the weakest gears as the following as :

$R_v = d_p / d_g = T_p / T_g = N_g / N_p = ¼$

$T_g = 64$

$yp = 0.154 - \frac{0.912}{Tp} = 0.154 - \frac{0.912}{16} = 0.097$

$yg = 0.154 - \frac{0.912}{Tg} = 0.154 - \frac{0.912}{64} = 0.13975$

3) $\sigma sp * yp = 83 * 0.097 = 8.05$ MN/ m^2 → weakest

$\sigma sg * yg = 103 * 0.13975 = 14.39$ MN/ m^2

The weakest gear is a pinion and the design on which is

done only , There fore lewis equation

4) $Ft = \sigma blan * b * pc * y$

$dp = m * Tp = 8 * 16 = 128$ mm

$V = \frac{\pi \, dp \, Np}{60} = \frac{\pi * \frac{128}{1000} * 600}{60}$ =4.02 m/ s

$K = \frac{3}{3+4.02} = 0.427$

$\sigma blan = \sigma s * k = 83 * 10^6 * 0.427$ =35.44 10^6 N/m

$p = Ft * v \rightarrow Ft = \frac{p}{V} = \frac{30.3 * 1000}{4.02} = 7537.3$ N

$P_{c\,=\,m}\pi$ = 25.13300 1000=0.025133 mm

$F_t = 6_{b*}\, b * P_c * y_p$ ----→ $b = F_t / b * P_c * y_p$

b= 7537.3/ 35.44*10^6*0.097 = 0.087m = 87mm

Ex/ 3. A bronze super gear pinion with static stress (6s =83 …..) Number of teeth (T_p= 16)and turn with speed (N= 600) drives a cast steel gear with a static stress (6=103M_N/m^2). If the pressure angle ($\emptyset = 20°$) $full$ depth involute profile of tooth , module (m=8)
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6 = 83MN/m^2 T_p = 16 , N_p= 600 rpm rv= ¼
6 = 103M_N/m^2 $\emptyset = 20°$, M = 8 , b = 90mm
the design must be done on the base of the weakest gears as the following as :

$R_v = d_p / d_g = T_p / T_g = N_g / N_p = ¼$

$F_t = 6_{b/an} * b * P_c * \ y_p$

$F_t = 35.44 * 10^6 * \frac{90}{1000} * \ 0.097$

$F_t = \ 7776\ N$

$P = F_t * V = 7776 * 4.02\ = 31259.5\ Watt$

$P = 31.26\ Kw$

Gear trains :
when two or more gears are in mixer with each other to transmit power from one shaft to another this combination
is called (**gear trains**) .
and it depends up on the velocity ratio and the relative position of axis of the shafts .

Type of gear trains :

1. Simple gear train.
2. Compound gear train .
3. Reverted gear train.
4. Epicyclic gear train .

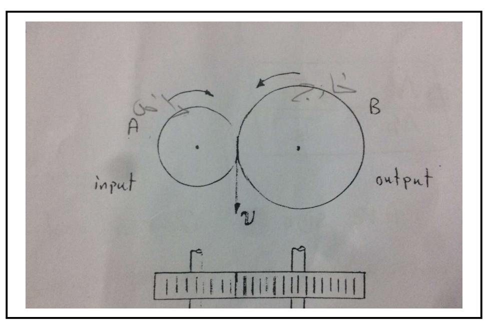
1. Simple gear train.

A/ External gear

$v = W_A \cdot r_A$

$v = W_B \cdot r_B$

$W_A \cdot r_A = W_B \cdot r_B$



$$\frac{W_B}{W_A} = \frac{\frac{2\pi N_B}{60}}{\frac{2\pi N_A}{60}} = \frac{-2r_A}{2r_B}$$

$$\boxed{\frac{N_B}{N_A} = \frac{-d_A}{d_B}}$$

$$:- m = \frac{d_A}{T_A} = \frac{d_B}{T_B} \quad \frac{d_A}{d_B} = \frac{T_A}{T_B}$$

$$\boxed{\frac{N_B}{N_A} = \frac{-T_A}{T_B}}$$ (-sign) gear

A and gear **B** rotating in opposite direction.

B/ internal gear

gear B has internal teeth

$$\frac{NB}{NA} = \frac{+TA}{TB}$$

+ve sign gear

A and gear **B** rotating inthe same direction.

A/ Idle gear trains :

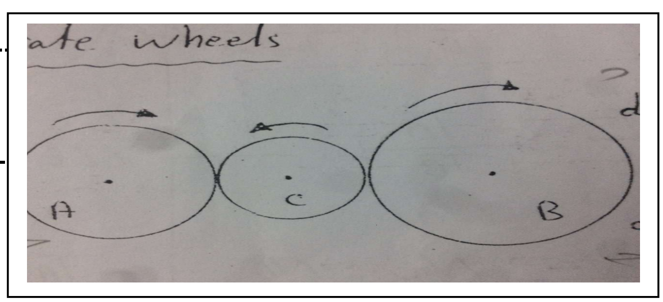
It is used when the distance between the two wheels is great and transmit ion

of the driver or follower is done by providing one or more intermediate wheels between them . if the number of the intermediate wheels are.

1. Odd numbers of intermediate wheels.

$$\frac{Nc}{NA} = \frac{-TA}{Tc}$$ ----------------

$$\frac{NB}{NC} = \frac{-T}{TB}$$ --------------



$$\frac{Nc}{NA}.\frac{NB}{NC}=\frac{-TA}{Tc}.\frac{-Tc}{TB}$$

$R=.\frac{NB}{NA}=\frac{+TA}{TB}$ +ve sign for odd number of idle gears.

2. <u>even number of intermediate wheels .</u>

drivers

$\frac{Nc}{NA}=\frac{-TA}{Tc}$ --------- i

$\frac{ND}{Nc}=\frac{-T}{TD}$ ---------- ii

$\frac{N\boldsymbol{B}}{ND}=\frac{-TD}{T\boldsymbol{B}}$ --------- iii

i*ii*iii

$$\frac{Nc}{NA}.\frac{ND}{Nc}.\frac{N\boldsymbol{B}}{ND}=\frac{-TA}{Tc}.\frac{-Tc}{TD}.\frac{-TD}{T\boldsymbol{B}}$$

$R_{ve}=\frac{N\boldsymbol{B}}{NA}=\frac{-TA}{TB}$ where + ve signs for even number of idle gears

where : N : rpm , T : number of teeth

d: diameter of gear Rv: velocity ratio

from above we see that the intermediate gears have have no effect on the velocity ratio and the named as ((dead gears))

<u>2. Compound gear train:</u>

in this case intermediate shaft has two wheels rigidly fixed to it , so that, they may have the same speed .

$\frac{Nc}{NA} = \frac{-TA}{TC}$ --------------------------- i

○ $N_c = N_D$ Compound

$\frac{NE}{ND} = \frac{-TD}{TE}$ -------------- ii

$\therefore N_E = N_F$

$N_B/N_F = -T_F/T_B$ ------------------- iii

i*ii*iii

$N_c/N_A \cdot N_E/N_D \cdot N_B/N_F = -T_A/T_C \cdot -T_D/T_E \cdot -T_F/T_B$

$$\mathbf{R_v} = \frac{NB}{NA} = \frac{-TA.TD.TF}{Tc.TE.TB}$$

<u>3.Reverted gear train: co-axial gear train:</u>

$N_c/ N_A = -T_A/T_c$ ----------------- i

$\therefore Nc = NA$ Compound

$N_B/N_D = -T_D/T_B$ …………………….. ii

i*ii

$N_c/N_A \,.\, N_B/N_D = - T_A / T_c \,.\, - T_D / T_B$

$R_v = N_B/N_A = + \frac{TA\ .TD}{Tc\ .TB}$

$\therefore\ m = d/T = 2r/T$

rArc = rDrB

$m_1.\ T_A /2 + m_1.\ T_c /2 = m_2.\ T_D /2\ \ m_2.\ T_B /2$

$M_1 (T_A + T_c) = M_2 (T_D + T_B)$

If $M_1 = M_2$

$T_A + T_c = T_D + T_B$

4. Epicycle gear train :

if the arm is
fixed the gear
train is simple
and wheel A can drive
wheel B or vice versa.
but , if wheel A is fixed
and the arm
is rotates then train
becomes an Epicycle gear train.

in Epicycle gear train , the axes of the shaft on which the gears are mounted more relative to axis

Tubular method :

Step No.	Condition of Motion	Revolution of elements		
		Arm C	Wheel **A**	Wheel **B**
1.	Arm fixed ; Wheel A rotates through +X rev.	0	+X	-X
2.	add + y rev.to all element .	+ y	+ y	+ y
3.	total motion	+ y	+X + y	+ y-X

If the wheel (A) is fixed -------→ NA = 0

NA = 0 = x + y x = -y

$N_B/N_c = \frac{y - X\,TA/TB}{y} = \frac{y - y\,(TA/TB)}{y} = \frac{y-(1+\frac{TA}{TB})}{y}$

$N_B/N_c = 1 + \frac{TA}{TB}$

2/ <u>Algerbric method</u>

In this method , the motion of each element of the epicycle train relative to the arm is set down in the form of equation

the number of equation depends upon the number of the element of gear train .

Let the arm (c) be fixed …..

the speed of the wheel (A) relative to the Arm (c) = N_A- Nc

the speed of the wheel (B) relative to the Arm (c) = N_B- Nc

the wheels A and B are meshing directly

N_B- Nc/ N_A- Nc = - T_A / T_B

Since The arm C is fixed Nc =0

$\frac{NB}{NA} = -\frac{TA}{TB}$

if the wheel A is fixed NA =0

$N_B - N_c / 0 - N_c = -\frac{TA}{TB}$

$N_B/N_c - 1 = +\frac{TA}{TB}$ ------→ $N_B/N_c = 1 + \frac{TA}{TB}$

<u>epicycle gear train :</u>

S: Sun Gear

P: plant gear

L:arm

A: annulus with
internal teeth

P: rotates freely

on a pin attached
to the arm L
L: rotates freely about
the axis of (S).

What is Ns /NL when A is fixed

Tabular Method

The Movement	NL	Ns	Np	NA
1. Fixed the arm , give $+X\ rev$ to wheel **P.**	0	-X T_p/ T_S	$+X$	$+X\ \frac{Tp}{TA}$
2. Add $+y\ rev\ \ to\ all$ wheels.	$+y$	$+y$	$+y$	$+y$
3. Sum	$+y$	$y-\ \ X\ Tp/Ts$	$X+\ y$	$y+X\ \ Tp/TA$

When **A** is fixed $\qquad$ N_A=0

$N_A = y + X\ \ T_p/T_A\ \ = \mathbf{0}$ $\qquad$ $y = X\,Tp/TA$

$$N_s/N_L = \frac{y - X\,Tp/Ts}{y} = \frac{-X\frac{Tp}{TA} - X\frac{Tp}{Ts}}{-X\frac{Tp}{TA}} = \frac{\frac{Tp}{TA} + \frac{Tp}{Ts}}{\frac{Tp}{TA}}$$

$$N_s/N_L = 1 + \frac{\frac{Tp}{Ts}}{\frac{Tp}{TA}} = 1 + \frac{Tp}{Ts}$$

$$N_s/N_L = 1 + \frac{Tp}{Ts}$$

π Algebra methed

When the arm is fixed

$$\frac{Np}{Ns} = -\frac{Ts}{Tp} \quad \dots\dots\dots..i$$

$$\frac{NA}{Np} = +\frac{Tp}{TA} \quad \dots\dots\dots\dots ii \qquad \text{or} \quad \frac{NA}{NS} = -\frac{Ts}{TA} \quad (i * ii)$$

When the arm is rotating

$$\frac{Np - NL}{Ns - NL} = \frac{Ts}{Tp}$$

$$\frac{NA - NL}{Np - NL} = \frac{Tp}{TA}$$

$$\boxed{\frac{N_A - N_L}{N_S - N_L} = \frac{-T_S}{T_A}}$$

When A is fixed

$$\frac{0 - NL}{Ns - N} = \frac{-Ts}{TA} \qquad \frac{Ns - NL}{NL} = \frac{TA}{Ts}$$

$$\frac{Ns}{NL} - 1 = \frac{TA}{Ts}$$

$$\frac{Ns}{NL} = 1 + \frac{TA}{Ts}$$

Ex1 - The gearing of machine tool is shown in fig below , the motor shaft is connected to wheel A and rotates at 975 RPM.
the gear wheels ((B,C,D , and E)) are fixed to the parallel shafts rotating together the final gear (F) is fixed on the out put shaft (G) what is the speed of (F) the number of teeth on each wheel are given below .

Gear	A	B	C	D	E	F
No.of teeth	20	50	25	75	26	65

Solution :

$R_v = N_f / N_A = -T_A.T_c.T_E/T_B.T_D.T_F$

$N_f / N_A = -20*25*26/ 50*75*65 = -13000/243750$

$N_f = (-0.0533) *975 = -52rpm$

$N_f = 52$ rpm in opposite direction of A .

EX2/ two parallel shaft , about **60**cm a part are to be connected by spur gear
Wheels , one shaft run with 360rpm
the other with 120rmp , design the wheels if the
circular pitch((25mm)).

Solution /

$N_2/N_1 = T_1/T_2 = d_1/d_2 = 120/ 360 = 1/3$

Center distance $C = d_1+d_2/ \mathbf{2} \cong 60cm$

$d_1+d_2 \cong 120$ --------------- (1)

$d_2= 3d_1$ --------------------------(2)

From (1) and (2) we get d2 =90 cm
d1= 30 cm

Number of teeth

$Pc_1 = \pi d_1/T_1$ ===========➔ $T_1 = \pi d_1/ Pc_1 = 37.7 \cong 38$

$Pc_2 = \pi d_2/T_2$ ===========➔ $T_2 = \pi d_2/ Pc_2 = 113.1 \cong (114)$

Fake number of teeth $T_1 = 38$

$T_1/ T_2 = 1/3$

$T_2 = 3\ T_1 = 114$ teeth

the exact diameter

$P_{c1} = \pi d_1 / T_1 \rightarrow d_1 = P_{c1} * T_1/\pi = 30.24$cm

$P_{c2} = \pi d_2 / T_2 \rightarrow d_2 = P_{c2} * T_2/\pi = 90.72$ cm

the exact center distance between Shaft s

$C = d_1 + d_2/2 = 60.48$ cm

Ex/ An epicycle gear consists of three wheels **A,B, and C** as shown below , wheel **A** has **72** internal teeth , **C** has 32 external teeth . the Wheel **B** gear with Both **A** and **C** , is carried on an arm which rotates about the Center of **A** at **18** RPM .
if the wheel **A** is Fixed , Determine the speed of the wheels
B and C .

Solution:

$T_A = 72 T_c = 32\ N_L = 18$ rpm

	N_L	N_c	N_B	N_A
1. Hold the arm give B + X rev.	0	$-X\frac{TB}{Tc}$	$+X$	$+X$
2. Give all gears $+y\ rev.$	$+y$	$+y$	$+y$	$+y$

3. Add	$+y$	$y - X\frac{TB}{Tc}$	$X + y$	$y + X$

Na = 0

$r_A = r_c + 2\,r_B$ $T_A = T_c + 2T_B$

72 = 32 + 2TB TB 20

$N_c = 18$ rpm

given

$N_A = 0$

$N_c = +y = 18$ $y = 18$

$N_A = y + X\frac{TB}{TA} = 0$ $0 = 18 + X\frac{20}{72}$

X= 64.8

N_B = X +y = - 64.8 + 18 = - 46.8 rpm

((46.8 rpm)) opposite direction of the arm L.

N_c = y - X $\frac{TB}{TC}$ = 18 – (-64.8) $\frac{20}{32}$ = + 58.5 rpm

= + 58.5 rpm in the direction of the arm L

<u>Gear box :</u>

<u>Design of simple gear box</u>

gear box is used to change constant speed of the motor into different required number of speed .

<u>the design of gear box may be done as follow as :</u>

Determine the number of steps and groups .

The required number of speed must be analysed in to its essential elements (mummer) in order to determine the number of steps , where every step consist of two groups one of them is moving and the other is fixed

-Draw kinematic diagram of dive .

-Construct table of speed .

<u>Ex</u>) Design a simple gear box lathe with six speed .

<u>Solution:-</u>

1) number of steps and groups is determined by analysing the number six in to its original

elements as follow as

$6 = 3 * 2$ → two steps

6

2 3

3 1

Every step consist of two gisups

A) sliding group (-) . B) fixed group (x) . as follow

Number of speed

2)kinematic diagram of drive .

130

Ex2) Design simple gear box as shown below and determine the speed on the π nd shaft , if 130

Ex2) Design simple gear box as shown below and determine the speed on the π nd shaft , if the speed ratio of the motor to the First shaft (I st)

Rv= $\frac{1}{3}$, Centre distance between the two shafts

(C = 20 cm) and diameter of gears as follow as (d1 = 20 cm , d4 = 16 cm , d5 = 8 cm) motor speed

(1200 rpm) .

Solution :-

$$\frac{NB}{NA} = \frac{1}{3} = Rv$$

NA = 1200 rpm

$$\frac{NB}{1200} = \frac{1}{3}$$

NB = 400 rpm

N1 = NB = 400 rpm   on the same shaft

$$C = 20\ cm = \frac{d1 + d2}{2}$$

$$\frac{20 + d2}{2} = 20\ cm$$

d2 = 20 cm

$$\frac{N2}{N1} = \frac{d1}{d2} \quad \rightarrow \quad \frac{N2}{400} = \frac{20}{20}$$

N2 = 400 rpm

N3 = NB = 400 rpm

$$C = 20\ cm = \frac{d3 + d4}{2} \quad \rightarrow \quad 20 = \frac{d3 + 16}{2}$$

d3 = 24 cm

$$\frac{N4}{N3} = \frac{d3}{4} \quad \rightarrow \quad \frac{N4}{400} = \frac{24}{16}$$

N4 = 600 rpm 2^{nd} speed

N5 = NB = 400 rpm

$$C = 20\ cm = \frac{d5 + d6}{2} \quad \rightarrow \quad 20 = \frac{8 + d6}{2}$$

d6 = 32 cm

$$\frac{N6}{N5} = \frac{d5}{d6} \quad \rightarrow \quad \frac{N6}{400} = \frac{8}{32}$$

N6 = 100 rpm 3rd speed

Ex3) Design simple gear box with nine speed .

Solution :-

1) number of steps $9 = 3 * 3$

	9
3	3
3	1

Number of groups

2)kinematic diagram of

Drive

1. Speed Chart

N_p

N_1 N_4 N_7 N_2 N_5 N_8 N_3 N_6 N_9

2. Speed Chart

1st step	2nd step	Speed No.	Value of the RPM
$A_1 - A_2$	$D_2 - D_1$	N1	$N_p * \frac{TA1}{TA2} * \frac{TD2}{TD1}$
$B_1 - B_2$	$D_2 - D_1$	N2	$N_p * \frac{TB1}{TB2} * \frac{TD2}{TD1}$
$C_1 - C_2$	$D_2 - D_1$	N3	$N_p * \frac{TC1}{TC2} * \frac{TD2}{TD1}$
$A_1 - A_2$	$E_2 - E_1$	N4	$N_p * \frac{TA1}{TA2} * \frac{TD2}{TD1}$
$B_1 - B_2$	$E_2 - E_1$	N5	$N_p * \frac{TA1}{TA2} * \frac{TE2}{TE1}$
$C_1 - C_2$	$E_2 - E_1$	N6	$N_p * \frac{TA1}{TA2} * \frac{TE2}{TE1}$
$A_1 - A_2$	$F_2 - F_1$	N7	$N_p * \frac{TA1}{TA2} * \frac{TF2}{TF1}$
$B_1 - B_2$	$F_2 - F_1$	N8	$N_p * \frac{TA1}{TA2} * \frac{TF2}{TF1}$
$C_1 - C_2$	$F_2 - F_1$	N9	$N_p * \frac{TA1}{TA2} * \frac{TF2}{TF1}$

Power Screw:
power screw is used to convert the turning motion into transverse motion . for example lead screw in leather .
Screw-jack. Presses ……. Etc…

I some Design the power screw turn and the nut is fixed in other the power screw is fixed and the nut is turn.

Type and terminology of power screw threads:

1. **Square thread**
this type is used to transmit motion in both directions with high efficiency and low redial presser on the nut.

2- **Acme thread** :
it is a modification of square thread . there is a slight inclination on both side of tooth to increase the cross section area of shear .

but the efficiency is reduce with respect to square thread also this inclination produce radical pressure …..

B- Buttress thread :
this type is used when the force acting on the screw axis is effect in one direction and so large this type large efficiency .

large across section area easy to cut
and used easy with a slip nut.

<u>Terminology of screw threads :</u>

Lead (L) : it is the distance by which the screw would advance relative to the nut in one revolution.

Single Thread Double Thread Tripled Thread

For single start thread $L = P$
For double start thread $L = 2P$
for triple start thread $L = 3P$

where as
L: lead
P : pitch

<u>Helix angle (X) :</u>
it is the relation
between the lead and the main diameter .

$$\text{Tan } X = \frac{L}{\pi dm}$$

$$X = \tan^{-1}\left(\frac{L}{\pi dm}\right)$$

turning moment (square thread) :

let/

di= root diameter (minor diameter)

dn = Major diameter of tread.

dm = main diameter of thread .

X = helix angle (degree)

M = coefficient of friction between power screw and nut .

$\boxed{dm = di + \frac{p}{2}}$

$\boxed{dn = di + p}$

Mc = coefficient friction for collar .

W = Axial load acts along screw axils.

P = pitch , B = friction angels ((degree))

Tan $\beta = M$ $\boxed{\beta = \tan^{-1} M}$

P = Tangential force

A: For raising the load

$$\tan(X+\beta) = \frac{Q}{W}$$

$$Q = W.\tan(X+\beta)$$

$$\frac{dm}{2}.Q = W.\frac{dm}{2}.\tan(X+\beta)$$

$$\boxed{\mathbf{T = W.\frac{dm}{2}.Tan(X+\beta)}}$$

for collar friction

$$T = W.\frac{dm}{2}.\text{Tan}\,(X + \beta) + Mc\,W\ rm$$

where :
T: applied torque (N.m.)

rm : means radius of collar friction (m)

Turning moment (Trapezoidal thread) :

$2\,\emptyset = 30\,°$

$M` = \frac{M}{cos\emptyset}$

Tan β` = M`

A: For raising the load

$$\mathbf{T = W.\frac{dm}{2}.\,Tan\,(X + \beta`)}$$

for collar friction

$$\mathbf{T = W.\frac{dm}{2}.\,Tan\,(X + \beta`) + Mc\,W\;rm}$$

B : for lowering the load

$$\mathbf{T = W.\frac{dm}{2}.\,Tan\,(\beta` - X)}$$

For collar friction

$$\mathbf{T = W.\frac{dm}{2}.\,Tan\,(\beta` - X) + Mc\,W\;rm}$$

Ex1 - A power screws has 0 major diameter (32 mm) ,

a pitch of (4mm) with double threads 3^{rd} is to be used with collar friction , ɥ = ɥc = 0.08 , rm = 20 mm , W = 6400 N square thread 3^{rd} .

1)The thread depth , thread width , mean or pitch diameter , minor diameter , and lead

2)The torque require to route the screw against

the load

3)The torque required to rotate tie screw with

the load

<u>Solution :-</u>

Do = 32 mm , p = 4mm , double star thread

$\mu c = \mu = 0.08$, rm = 20 mm , W = 6400 N , square thread

1) thread depth = $\frac{p}{2} = 2\ mm$

thread width = $\frac{p}{2} = 2\ mm$

Mean diameter dm = do - p/2 = 30mm

minor diameter di = do - p = 28 mm

lead = 2 ∗ p = 8 mm

2) $\beta = \tan^{-1} \mu = 4.57^o$

$\alpha = \tan^{-1}\left(\frac{L}{\pi dm}\right) = 4.85^o$

$T = W * \frac{dm}{2} * \tan(\alpha + \beta) + \mu c\, W\, rm$

$= 6400 * \frac{\frac{30}{1000}}{2} * tan(\alpha + \beta) + 0.08 * 6400 * \frac{20}{100}$

T = 15.94 + 10.24 = 26.24 N∗m

3)T = W ∗ dm/2 ∗ $tan(\alpha - \beta) + \mu c\, W\, rm$

$= 6400 * \frac{\frac{30}{1000}}{2} * tan(\alpha - \beta) + 0.08 * 6400 * \frac{20}{100}$

= - 0.466 + 10.24

$T = 9.77\ N*m$