Matrices

Lecture 1

Dr. Alaa Alasadi.

Matrix

A matrix is every set of numbers or terms arranged in a rectangular shape, forming rows and columns.

Each number in a matrix is an **element**. One element is distinguished from another by its position, that is to say, the row and column to which it belongs.

The number of rows and columns of a matrix is called the **dimension of a matrix**. Thus, a matrix is of dimension: 2x4, 3x2, 2x5,... If the matrix has the same number of rows and columns, is said to be of order: 2, 3, ...

The set of matrices of ***m*** rows and ***n*** columns is denoted by $\boldsymbol{A}_{\mathbf{mxn}}$ or ($\mathbf{a}_{\mathbf{ij}}$), and any element within the matrix is in row ***i*** in column ***j***, for $\mathbf{a}_{\mathbf{ij}}$.

Two matrices are equal when they have the same dimension and equal elements which occupy the same place in both.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

Types of Matrices

In this section, you will find exercises and worksheets to review the theory of matrices in Algebra. A **matrix is a rectangular array of numbers** or other mathematical objects for which operations such as addition and multiplication are defined

Row Matrix

A row matrix is formed by a single row.

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$$

Column Matrix

A column matrix is formed by a single column.

$$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

Zero Matrix

In a zero matrix, all the elements are zeros.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Rectangular Matrix

A rectangular matrix is formed by a different number of rows and columns, and its dimension is noted as: **mxn**.

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Square Matrix

A square matrix is formed by the same number of rows and columns.
The elements of the form aii constitute the principal diagonal.
The secondary diagonal is formed by the elements with i+j = n+1.

$$\begin{bmatrix} 2 & -3 & 6 \\ 0 & 2 & 7 \\ -1 & 4 & 3 \end{bmatrix}$$

Upper Triangular Matrix

In an upper triangular matrix, the elements located below the diagonal are zeros.

$$\begin{bmatrix} 1 & 7 & 5 \\ 0 & 6 & 3 \\ 0 & 0 & 8 \end{bmatrix}$$

Lower Triangular Matrix

In a lower triangular matrix, the elements above the diagonal are zeros.

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 6 & 0 \\ 4 & 5 & 8 \end{bmatrix}$$

Diagonal Matrix

In a diagonal matrix, all the elements above and below the diagonal are zeros.

$$\begin{bmatrix} 2 & o & o \\ 0 & -1 & o \\ o & o & 3 \end{bmatrix}$$

Scalar Matrix

A scalar matrix is a diagonal matrix in which the diagonal elements are equal.

$$\begin{bmatrix} 2 & o & o \\ 0 & 2 & o \\ o & o & 2 \end{bmatrix}$$

Identity Matrix

An identity matrix is a diagonal matrix in which the diagonal elements are equal to 1.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Mathematics

Matrices

Lecture 2

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Addition of matrices

Given two matrices of the same dimension, $\mathbf{A = (a_{ij})}$ and $\mathbf{B = (b_{ij})}$, the matrix sum is defined as: $\mathbf{A + B = (a_{ij} + b_{ij})}$. That is, the resultant matrix's elements are obtained by adding the elements of the two matrices that occupy the same position.

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 0 & 0 \\ 5 & 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 2+1 & 0+0 & 1+1 \\ 3+1 & 0+2 & 0+1 \\ 5+1 & 1+1 & 1+0 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 2 \\ 4 & 2 & 1 \\ 6 & 2 & 1 \end{pmatrix}$$

$$A - B = \begin{pmatrix} 2-1 & 0-0 & 1-1 \\ 3-1 & 0-2 & 0-1 \\ 5-1 & 1-1 & 1-0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -2 & -1 \\ 4 & 0 & 1 \end{pmatrix}$$

Properties of the Addition of Matrices

- The sum of two matrices of dimension **m x n** is another matrix of dimension **m x n.**

Associative:

A + (B + C) = (A + B) + C

Given:

$$A=\begin{bmatrix} 2 & 4 & 6 \\ 4 & 0 & 3 \\ -2 & 0 & 2 \end{bmatrix} \qquad B=\begin{bmatrix} -2 & 3 & -1 \\ -1 & 1 & 7 \\ 6 & -1 & 1 \end{bmatrix} \qquad C=\begin{bmatrix} 7 & 5 & -1 \\ 4 & 3 & 5 \\ -4 & 0 & 6 \end{bmatrix}$$

Additive identity:

A + 0 = A

Where **0** is the zero matrix of the same dimension.

$$A=\begin{bmatrix} -2 & 3 & -1 \\ -1 & 1 & 7 \\ 6 & -1 & 1 \end{bmatrix} \qquad Z=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A+Z=\begin{bmatrix} -2+0 & 3+0 & -1+0 \\ -1+0 & 1+0 & 7+0 \\ 6+0 & -1+0 & 1+0 \end{bmatrix}=\begin{bmatrix} -2 & 3 & -1 \\ -1 & 1 & 7 \\ 6 & -1 & 1 \end{bmatrix}$$

Additive inverse:

A + (−A) = O where **−A= -1[A]**

The opposite matrix has each of its elements change sign.

$$A=\begin{bmatrix} -2 & 3 & -1 \\ -1 & 1 & 7 \\ 6 & -1 & 1 \end{bmatrix} \qquad -1XA=\begin{bmatrix} 2 & -3 & 1 \\ 1 & -1 & -7 \\ -6 & 1 & -1 \end{bmatrix}$$

$$So\ A+(-A)=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Commutative:

A + B = B + A

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 0 & 0 \\ 5 & 1 & 1 \end{pmatrix}; \qquad B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Given the matrices:

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 0 & 0 \\ 5 & 1 & 1 \end{pmatrix}; \qquad B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Calculate: **A + B; A - B**

$$A + B = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 0 & 0 \\ 5 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2+1 & 0+0 & 1+1 \\ 3+1 & 0+2 & 0+1 \\ 5+1 & 1+1 & 1+0 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 2 \\ 4 & 2 & 1 \\ 6 & 2 & 1 \end{pmatrix}$$

$$A - B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 1 \\ 3 & 0 & 0 \\ 5 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2-1 & 0-0 & 1-1 \\ 3-1 & 0-2 & 0-1 \\ 5-1 & 1-1 & 1-0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -2 & -1 \\ 4 & 0 & 1 \end{pmatrix}$$

Transpose Matrix

Given matrix A, the transpose of matrix A is another matrix where the elements in the columns and rows have switched. In other words, the rows become the columns and the columns become the rows.

$$A = \begin{bmatrix} 2 & 5 & -1 \\ 4 & 1 & 3 \\ -2 & 0 & 1 \end{bmatrix} \qquad A^t = \begin{bmatrix} 2 & 4 & -2 \\ 5 & 1 & 0 \\ -1 & 3 & 1 \end{bmatrix}$$

Properties of Transpose Matrix

$(A^t)^t = A$

$$A=\begin{bmatrix} 2 & 5 & -1 \\ 5 & 1 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A^t=\begin{bmatrix} 2 & 5 & -1 \\ 5 & 1 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$(A^t)^t=\begin{bmatrix} 2 & 5 & -1 \\ 5 & 1 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A=\begin{bmatrix} 2 & 5 & -1 \\ 5 & 1 & 3 \\ -1 & 3 & 1 \end{bmatrix} \qquad B=\begin{bmatrix} 2 & 5 & -1 \\ 5 & 1 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

$(A + B)^t = A^t + B^t$

$(\alpha \cdot A)^t = \alpha \cdot A^t$

$(A \cdot B)^t = B^t \cdot A^t$

Symmetric Matrix

A symmetric matrix is a square matrix that verifies:

$$\boldsymbol{A = A^t}.$$

$$A=\begin{bmatrix} 2 & 5 & -1 \\ 5 & 1 & 3 \\ -1 & 3 & 1 \end{bmatrix} \qquad A^t=\begin{bmatrix} 2 & 5 & -1 \\ 5 & 1 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

Antisymmetric Matrix

An antisymmetric matrix is a square matrix that verifies:

$$\boldsymbol{A = -A^t.}$$

$$A=\begin{bmatrix} 2 & 5 & 1 \\ 5 & 1 & 3 \\ 1 & 3 & 1 \end{bmatrix} \qquad A^t=\begin{bmatrix} 2 & 5 & -1 \\ 5 & 1 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

Next week:

Multiplying Matrices

Thanks for your attention

Matrices

Lecture 3

Multiplying Matrices

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Two matrices A and B can be multiplied together if the number of columns of A is equal to the number of rows of B.

$$\mathbf{M_{m \times n} \times M_{n \times p} = M_{m \times p}}$$

The element, $\mathbf{c_{ij}}$, of the product matrix is obtained by multiplying every element in row $\mathbf{i}$ of matrix *A* by each element of column $\mathbf{j}$ of matrix ***B*** and then adding them together.

$$A \cdot B = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 0 & 0 \\ 5 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 2\cdot 1 + 0\cdot 1 + 1\cdot 1 & 2\cdot 0 + 0\cdot 2 + 1\cdot 1 & 2\cdot 1 + 0\cdot 1 + 1\cdot 0 \\ 3\cdot 1 + 0\cdot 1 + 0\cdot 1 & 3\cdot 0 + 0\cdot 2 + 0\cdot 1 & 3\cdot 1 + 0\cdot 1 + 0\cdot 0 \\ 5\cdot 1 + 1\cdot 1 + 1\cdot 1 & 5\cdot 0 + 1\cdot 2 + 1\cdot 1 & 5\cdot 1 + 1\cdot 1 + 1\cdot 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 3 & 1 & 2 \\ 3 & 0 & 3 \\ 7 & 3 & 6 \end{pmatrix}$$

Properties of Matrix Multiplication

Associative:

$$\mathbf{A \cdot (B \cdot C) = (A \cdot B) \cdot C}$$

Multiplicative Identity

$$\mathbf{A \cdot I = A}$$

Where ***I*** is the identity matrix with the same order as matrix *A*.

Not Commutative:

$$\mathbf{A \cdot B \neq B \cdot A}$$

Distributive:

$$\mathbf{A \cdot (B + C) = A \cdot B + A \cdot C}$$

Given the matrices:

$$A=\begin{bmatrix}2 & 0 & 1\\ 3 & 0 & 0\\ 5 & 1 & 1\end{bmatrix} \qquad B=\begin{bmatrix}1 & 0 & 1\\ 1 & 2 & 1\\ 1 & 1 & 0\end{bmatrix}$$

Calculate:

A x B; B x A

$$A\cdot B=\begin{pmatrix}2 & 0 & 1\\ 3 & 0 & 0\\ 5 & 1 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0 & 1\\ 1 & 2 & 1\\ 1 & 1 & 0\end{pmatrix}=\begin{pmatrix}2\cdot1+0\cdot1+1\cdot1 & 2\cdot0+0\cdot2+1\cdot1 & 2\cdot1+0\cdot1+1\cdot0\\ 3\cdot1+0\cdot1+0\cdot1 & 3\cdot0+0\cdot2+0\cdot1 & 3\cdot1+0\cdot1+0\cdot0\\ 5\cdot1+1\cdot1+1\cdot1 & 5\cdot0+1\cdot2+1\cdot1 & 5\cdot1+1\cdot1+1\cdot0\end{pmatrix}=\begin{bmatrix}3 & 1 & 2\\ 3 & 0 & 3\\ 7 & 3 & 6\end{bmatrix}$$

$$B\cdot A=\begin{pmatrix}1 & 0 & 1\\ 1 & 2 & 1\\ 1 & 1 & 0\end{pmatrix}\cdot\begin{pmatrix}2 & 0 & 1\\ 3 & 0 & 0\\ 5 & 1 & 1\end{pmatrix}=\begin{pmatrix}1\cdot2+0\cdot3+1\cdot5 & 1\cdot0+0\cdot0+1\cdot1 & 1\cdot1+0\cdot0+1\cdot1\\ 1\cdot2+2\cdot3+1\cdot5 & 1\cdot0+2\cdot0+1\cdot1 & 1\cdot1+2\cdot0+1\cdot1\\ 1\cdot2+1\cdot3+0\cdot5 & 1\cdot0+1\cdot0+0\cdot1 & 1\cdot1+1\cdot0+0\cdot1\end{pmatrix}=\begin{bmatrix}7 & 1 & 2\\ 13 & 1 & 2\\ 5 & 0 & 1\end{bmatrix}$$

Given the matrices:

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & -2 \\ 0 & 2 \\ -2 & 0 \end{pmatrix}$$

Example 1.

$(A^t \cdot B) \cdot C$

$(A^t_{3 \times 2} \cdot B_{2 \times 2}) \cdot C_{3 \times 2} = (A^t \cdot B)_{3 \times 2} \cdot C_{3 \times 2}$

The multiplication is not possible because the number of columns, $(A^t \cdot B)$ does not coincide with the numbers of rows of C.

Example 2.

$(B \cdot C^t) \cdot A^t$

$(B_{2 \times 2} \cdot C^t_{2 \times 3}) \cdot A^t_{3 \times 2} = (B \cdot C)_{2 \times 3} \cdot A^t_{3 \times 2} =$

$=(B \cdot C^t \cdot A^t)_{2 \times 2}$

Determine the dimension of $\mathbf{M}$ so that the multiplication is possible: $\mathbf{A} \cdot \mathbf{M} \cdot \mathbf{C}$

$\mathbf{A}_{3 \times 2} \cdot \mathbf{M}_{m \times n} \cdot \mathbf{C}_{3 \times 2}$ $\mathbf{m = 2}$

Determine the dimension of $\mathbf{M}$ if $\mathbf{C}^t \cdot \mathbf{M}$ is a square matrix.

$\mathbf{C}^t_{2 \times 3} \cdot \mathbf{M}_{m \times n}$ $\mathbf{m = 3}$ $\mathbf{n = 3}$

Determinants

Lecture 4

Definition of Determinants

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Determinants

Suppose A is an *n* x *n* matrix. Associated with A is a *number* called the **determinant of A** and is denoted by *det A or |A|*. Symbolically, we distinguish a matrix A from the *determinant of A* by replacing the parentheses by vertical bars:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

and

$$det A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

and

$$det A = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix}$$

Thus, every *square* matrix **A**, is assigned a particular scalar quantity called the **determinant of A**, denoted by *det A* or |A|.

A *determinant* of an *n* x *n* matrix is said to be a determinant of *order n*.

Let's begin by defining the determinants of 1 x 1, 2 x 2 , and 3 x 3 matrices.

The Determinant of Order One

For a 1 x 1 matrix A = (a) we have det A = |a| = a.

For example, if A = (-5), then *det A* = |-5| = -5.

Note: In this case, the vertical bars || around a number do not mean the absolute value of the number.

The Determinant of Order Two

The determinant of a 2 x 2 matrix is said to be a determinant of order 2.

Consider matrix A:

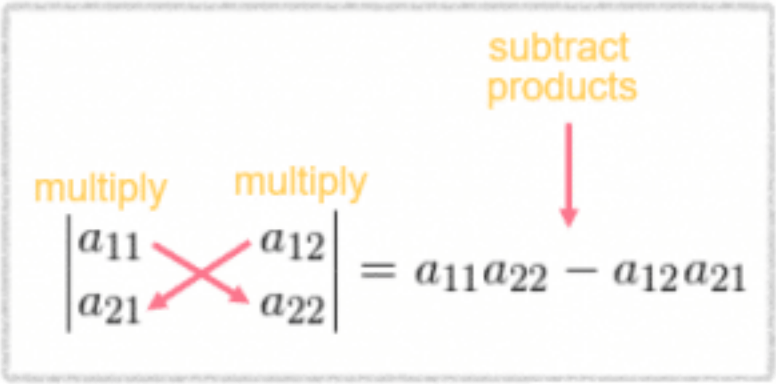
$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

The determinant of

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

is the number

$$det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$



As a mnemonic, a determinant of order 2 is thought to be the product of the main diagonal entries of **A** minus the product of the other diagonal entries.

For example, if matrix A is the 2 x 2 matrix

$$A = \begin{bmatrix} 5 & -7 \\ 2 & 9 \end{bmatrix}$$

then det. A is

$$|A =| \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

So

$$|A| = (5 * 9) - (-7 * 2) = 45 - (-14) = 59$$

The Determinant of Order Three

The determinant of a 3 x 3 matrix is said to be a determinant of order 3.
Consider matrix A:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The determinant of

$$det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

We can calculate the determinant of order three as following by minores method:

$$detA = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{12}\left[-\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}\right] + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$detA = a_{11}(a_{22}a_{33} - a_{23}a_{32}) + a_{12}(-a_{21}a_{33} + a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Example:
Evaluate the determinant of

$$A = \begin{bmatrix} 2 & 4 & 7 \\ 6 & 0 & 3 \\ 1 & 5 & 3 \end{bmatrix}$$

$$detA = \begin{vmatrix} 2 & 4 & 7 \\ 6 & 0 & 3 \\ 1 & 5 & 3 \end{vmatrix}$$

where the dashed lines indicate the row and column that are deleted. Thus,

$$detA = 2(-1)^{1+1}\begin{vmatrix} 0 & 3 \\ 5 & 3 \end{vmatrix} + 4(-1)^{1+2}\begin{vmatrix} 6 & 3 \\ 1 & 3 \end{vmatrix} + 7(-1)^{1+3}\begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix}$$

$$= 2[0(3) - 3(5)] - 4[6(3) - 3(1)] + 7[6(5) - 0(1)] = 120$$

Example 2:
Evaluate the determinant of

$$A = \begin{bmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{bmatrix}$$

Solution: Since there are two zeros in the third column, we expand by cofactors of that column:

$$detA = \begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix}$$

$$= (-7)(-1)^{2+3} \begin{vmatrix} 6 & 5 & 0 \\ 1 & 8 & 7 \\ -2 & 4 & 0 \end{vmatrix} = (-7)(-1)^{2+3} \begin{vmatrix} 6 & 5 \\ -2 & 4 \end{vmatrix}$$

$$= 7[6(4) - 5(-2)] = 238$$

Determinants

Lecture 5

Calculation of determinant

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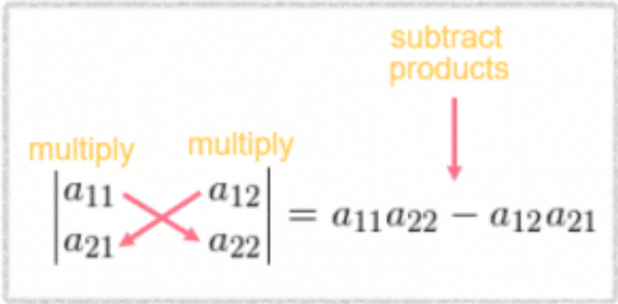
The Determinant of Order Two



For example, if matrix A is the 2 x 2 matrix

$$A = \begin{bmatrix} 5 & -7 \\ 2 & 9 \end{bmatrix}$$

then det A is:

$$|A = | \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

So

$$|A| = (5 * 9) - (-7 * 2) = 59$$

Minors method

We can calculate the determinants by minors method:

Minor:

An element, **a_{ij}**, to the value of the determinant of order n − 1, obtained by deleting the row ***i*** and the column ***j*** in the matrix is called a **minor**.

Now, we select the element of (a_{22}) which have value 5, by deleting its row and column will be:

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & \boxed{5} & 4 \\ 3 & 6 & 2 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$$

The Determinant of Order Three

We can calculate the determinant of order three as following:

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\det A = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{12} \left[- \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \right] + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\det A = a_{11}(a_{22}a_{33} - a_{23}a_{32}) + a_{12}(-a_{21}a_{33} + a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

- We can determine the sign of element by

$$(-1)^{i+j}$$

where i is the row of element and j is column

Example:
Evaluate the determinant of

$$A=\begin{vmatrix} 1 & -2 & 5 \\ -3 & 1 & 2 \\ 3 & -4 & -1 \end{vmatrix}$$

We select the first row:

$(-1)^{i+j}$

$$A = (-1)^{1+1}(1)\begin{vmatrix} 1 & 2 \\ -4 & -1 \end{vmatrix} + (-1)^{1+2}(-2)\begin{vmatrix} -3 & 2 \\ 3 & -1 \end{vmatrix} + (-1)^{1+3}(5)\begin{vmatrix} -3 & 1 \\ 3 & -4 \end{vmatrix}$$

$$A = 1\,[(1*(-1)-(2*(-4))] + (-1)(-2)[((-3)*(-1)-(2*3)] + 5[(-3)*(-4)-(1*3)]$$

$$A = [(-1+8)+2(3-6)+5(12-3)]$$

A=$7-6-45$=46

Other method to calculate determinant

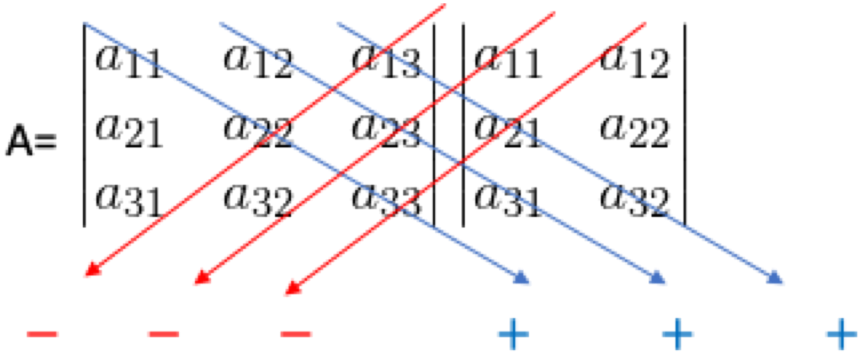
We can calculate the determinants by special method,
this method can be apply by:

1- Repeat a first and second column beside the main determinant.
2- Then draw three lines along main diagonals with plus signs then three lines along second diagonals with minus signs.
3- Multiply the elements of each diagonal each other.
4- Calculate the summation value of multiplications.

This value is the determinant.

Special method to calculate the value of determinant of order 3

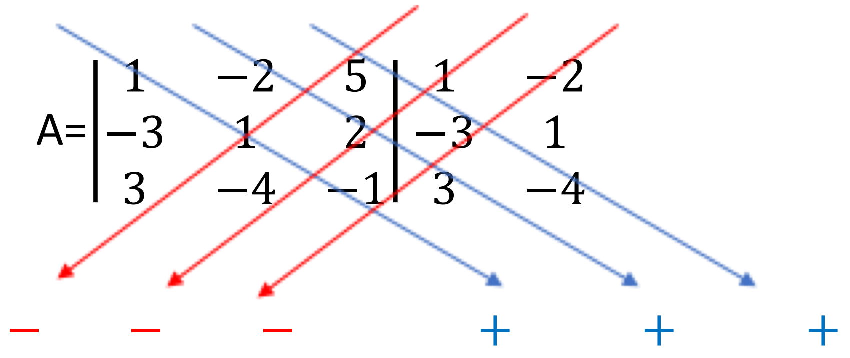
$$det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$



$$A = (a_{11} * a_{22} * a_{33}) + (a_{12} * a_{23} * a_{31}) + (a_{13} * a_{21} * a_{32}) - (a_{13} * a_{22} * a_{31}) - (a_{11} * a_{23} * a_{32}) - (a_{12} * a_{21} * a_{33})$$

Example:

Evaluate the determinant of $A=\begin{vmatrix} 1 & -2 & 5 \\ -3 & 1 & 2 \\ 3 & -4 & -1 \end{vmatrix}$



$$A = (1*1*(-1)) + (-2*2*3) + (5*-3*-4) - (5*1*3) - (1*2*(-4)) - ((-2)*(-3)*(-1))$$

A= 46

Ex2:

Find the value of following determinant

$$A=\begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix}$$

$$=\begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix}\begin{matrix} 6 & 5 \\ -1 & 8 \\ -2 & 4 \end{matrix}$$

A=0+70+0-0+168-0

A=-238

Properties of determinants:

١- محدد المصفوفة المبدولة يساوي المحدد المصفوفة الاصلية:

$$A=\begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix}$$

$$A^t=\begin{vmatrix} 6 & -1 & -2 \\ 5 & 8 & 4 \\ 0 & -7 & 0 \end{vmatrix}\begin{matrix} 6 & -1 \\ 5 & 8 \\ 0 & -7 \end{matrix}\Bigg|$$

A^t=(6*8*0)+(-1*4*0)+(-2*5*-7)-((-2*8*0)-(6*4*-7)-(-1*5*0)

A^t=(0+0+70-0+168+0)
=238

٢-اذا كان جميع عناصر صف او عمود اصفار فان قيمة المحدد تساوي صفر :

$$A=\begin{vmatrix} 6 & 3 & 0 \\ -1 & -3 & 0 \\ -2 & 4 & 0 \end{vmatrix}\begin{matrix} 6 & 3 \\ -1 & -3 \\ -2 & 4 \end{matrix}$$

$$A=0+0+0-0-0-0=0$$

٣-اذا ابدل صفان او عمودان كل محل الاخر تتغير إشارة المحدد :

$$A=\begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix} \quad =238$$

$$A=\begin{vmatrix} 5 & 6 & 0 \\ 8 & -1 & -7 \\ 4 & -2 & 0 \end{vmatrix}\begin{matrix} 5 & 6 \\ 8 & -1 \\ 4 & -2 \end{matrix}\Bigg|$$

A=(0-168+0)+(0-70-0)

A=-238

٤-اذا تطابقت عناصر صفان او عمودان فان قيمة المحدد تساوي صفر :

$$A=\begin{vmatrix} 3 & 4 & 0 \\ -1 & 2 & -7 \\ -1 & 2 & -7 \end{vmatrix}\begin{matrix} 3 & 4 \\ -1 & 2 \\ -1 & 2 \end{matrix}$$

A=(-42+28+0)+(0+42-28)

A=0

٥-اذا ضرب عناصر صف او عمود بثابت مثل (x) فان قيمة المحدد الجديد تساوي فان قيمة المحدد تساوي (A*x:

X=2

$$A=\begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix} = 238$$

A*X=238*2=476

مثلا نضرب العمود الثالث بالثابت الجديد 2

$$A*2=\begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -14 \\ -2 & 4 & 0 \end{vmatrix}\begin{vmatrix} 6 & 5 \\ -1 & 8 \\ -2 & 4 \end{vmatrix}$$

A*X=0+140+0+0+336-0)=476

Next week

- **The Determinant of Order four**

شكرا للإصغاء

Determinants

Lecture 6

Properties of determinants

Dr. Alaa Alasadi.

١ - محدد المصفوفة المبدولة يساوي المحدد المصفوفة الاصلية:

$$|A| = |A^t|$$

$$A = \begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix} = 238$$

$$A^t = \left|\begin{matrix} 6 & -1 & -2 \\ 5 & 8 & 4 \\ 0 & -7 & 0 \end{matrix}\middle|\begin{matrix} 6 & -1 \\ 5 & 8 \\ 0 & -7 \end{matrix}\right|$$

A^t=(6*8*0)+(-1*4*0)+(-2*5*-7)-(-2*8*0)-(6*4*-7)-(-1*5*0)

A^t=0+0+70-0+168-0= 238=A

٢-اذا كان جميع عناصر صف او عمود اصفار فان قيمة المحدد تساوي صفر :

$$A = \left|\begin{array}{ccc|cc} 4 & 2 & 0 & 4 & 2 \\ -9 & 5 & 0 & -9 & 5 \\ -6 & -3 & 0 & -6 & -3 \end{array}\right|$$

A=0+0+0-0-0-0=0

٣-اذا ابدل صفان او عمودان كل محل الاخرفان قيمة المحدد تبقى نفسها مع تغير إشارته :

Ex:

$$A = \begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix} = 238$$

$$A = \begin{vmatrix} 0 & 5 & 6 \\ -7 & 8 & -1 \\ 0 & 4 & -2 \end{vmatrix} \begin{matrix} 0 & 5 \\ -7 & 8 \\ 0 & 4 \end{matrix}$$

A=0+0-168-0-0-70=-238

Home work

احسب محدد المصفوفة ادناه ثم استبدل الصف الثاني بالصف الثالث واحسب المحدد الجديد

$$A = \begin{vmatrix} -2 & 4 & 0 \\ -1 & 8 & -7 \\ 6 & 5 & 0 \end{vmatrix}$$

٤-اذا تطابقت عناصر صفان او عمودان فان قيمة المحدد تساوي صفر :

$$A = \begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -1 & 8 & -7 \end{vmatrix} \begin{matrix} 6 & 5 \\ -1 & 8 \\ -1 & 8 \end{matrix}$$

A=(-336+35+0)+(0+336-35)=0

Home work

$$A = \begin{vmatrix} 6 & 5 & 5 \\ -1 & 8 & 8 \\ -2 & 4 & 4 \end{vmatrix}$$

٥-اذا ضرب عناصر صف او عمود بثابت مثل X فان قيمة المحدد الجديد تساوي فان قيمة المحدد تساوي A*X :

Ex:

Let X is constant

X=4

$$A = \begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix} = 238$$

A*X=238*4=952

ندخل الثابتX على العمود الثاني

$$A * X = \begin{vmatrix} 6 & 20 & 0 \\ -1 & 32 & -7 \\ -2 & 16 & 0 \end{vmatrix} \begin{matrix} 6 & 20 \\ -1 & 32 \\ -2 & 16 \end{matrix} \Bigg|$$

A*X=0+280+0-0+672-0=952

Home work

ادخل الثابت على الصف الثالث

$$A = \begin{vmatrix} 2 & 5 & -1 \\ -1 & 4 & -7 \\ -2 & -3 & 0 \end{vmatrix}$$

X=-3

٦-اذا ضرب عناصر صف او عمود بثابت مثل X ,واضيفت الى صف او عمود اخر فان قيمة المحدد الجديد تبقى ثابتة:

$$A=\begin{vmatrix}6&5&0\\-1&8&-7\\-2&4&0\end{vmatrix}=238$$

مثلا, نضرب العمود الأول (2) ونضيفه الى العمود الثاني في

$$A=\left|\begin{matrix}6&17&0\\-1&6&-7\\-2&0&0\end{matrix}\right|\left.\begin{matrix}6&17\\-1&6\\-2&0\end{matrix}\right|$$

A=0+238+0-0-0-0=238

نضرب الصف الثالث في 3 ونضيفه الصف الأول

$$A=\left|\begin{matrix}0&17&0\\-1&6&-7\\-2&0&0\end{matrix}\right|\left.\begin{matrix}0&17\\-1&6\\-2&0\end{matrix}\right|$$

A=0+238+0-0-0-0=238

نضرب العمود الثاني في 4 ونضيفه العمود الثالث

$$A = \begin{vmatrix} 0 & 17 & 0 \\ -1 & 6 & -7 \\ -2 & 0 & 0 \end{vmatrix}$$

$$A = \begin{vmatrix} 0 & 17 & 68 \\ -1 & 6 & 17 \\ -2 & 0 & 0 \end{vmatrix}\begin{matrix} 0 & 17 \\ -1 & 6 \\ -2 & 0 \end{matrix}\Bigg|$$

A=0-578-0+816-0-0=238

$$A = \begin{vmatrix} 6 & 5 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix} = 238$$

$$A = \begin{vmatrix} 5 & 6 & 0 \\ -1 & 8 & -7 \\ -2 & 4 & 0 \end{vmatrix}\begin{matrix} 5 & 6 \\ -1 & 8 \\ -2 & 4 \end{matrix}\Bigg|$$

A=0+84+0-0+140-0=224

Next week

- **The Determinant of Order four**

شكرا للإصغاء

Determinants

Lecture 7

Calculation of 4x4 Determinant

Dr. Alaa Alasadi.

We will use the same approach that we saw in the last lecture, where we expanded a 3×3 determinant.

Going down the first column or first row, we find the cofactors of each element and then multiply each element by its cofactor.

But here we will use first column:

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

$(-1)^{i+j}$ $(-1)^{i+j}$ $(-1)^{i+j}$ $(-1)^{i+j}$

$$|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix} + a_{31}\begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} - a_{41}\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix}$$

To get the final answer, we expand out these 3×3 determinants, multiply then simplify.

The first step is to find the cofactors of each of the elements in the first column. We then multiply by the elements of the first row and assign plus and minus in the order:
plus, minus, plus, minus

By choosing first column

Ex1:

$$|A| = \begin{vmatrix} 7 & 4 & 2 & 0 \\ 6 & 3 & -1 & 2 \\ 4 & 6 & 2 & 5 \\ 8 & 2 & -7 & 1 \end{vmatrix}$$

$(-1)^{i+j}$

$$|A| = 7\begin{vmatrix} 3 & -1 & 2 \\ 6 & 2 & 5 \\ 2 & -7 & 1 \end{vmatrix} - 6\begin{vmatrix} 4 & 2 & 0 \\ 6 & 2 & 5 \\ 2 & -7 & 1 \end{vmatrix} + 4\begin{vmatrix} 4 & 2 & 0 \\ 3 & -1 & 2 \\ 2 & -7 & 1 \end{vmatrix} - 8\begin{vmatrix} 4 & 2 & 0 \\ 3 & -1 & 2 \\ 6 & 2 & 5 \end{vmatrix}$$

Now, we expand out each of those 3 × 3 cofactors using the method that we saw before:

$$\begin{vmatrix} 3 & -1 & 2 \\ 6 & 2 & 5 \\ 2 & -7 & 1 \end{vmatrix} = 15$$

$$\begin{vmatrix} 4 & 2 & 0 \\ 6 & 2 & 5 \\ 2 & -7 & 1 \end{vmatrix} = 156$$

$$\begin{vmatrix} 4 & 2 & 0 \\ 3 & -1 & 2 \\ 2 & -7 & 1 \end{vmatrix} = 54$$

$$\begin{vmatrix} 4 & 2 & 0 \\ 3 & -1 & 2 \\ 6 & 2 & 5 \end{vmatrix} = -42$$

Home work:

Find the determinant of each minores by minores method and special method.

Now, putting it all together, we have:

$$\begin{vmatrix} 7 & 4 & 2 & 0 \\ 6 & 3 & -1 & 2 \\ 4 & 6 & 2 & 5 \\ 8 & 2 & -7 & 1 \end{vmatrix}$$

$$= 7 \times 15 - 6 \times 156 + 4 \times 54 - 8 \times -42$$

$$= -279$$

مثلا نختار العمود الرابع

$$|A|=\begin{vmatrix} 2 & 1 & 4 & -1 \\ -2 & 0 & 3 & -1 \\ 0 & 3 & -2 & 2 \\ 3 & 2 & -1 & 0 \end{vmatrix}$$

$(-1)^{i+j}$

$$|A|=-1(-1)^{1+4}\begin{vmatrix} -2 & 0 & 3 \\ 0 & 3 & -2 \\ 3 & 2 & -1 \end{vmatrix}-1\begin{vmatrix} 2 & 1 & 4 \\ 0 & 3 & -2 \\ 3 & 2 & -1 \end{vmatrix}-2\begin{vmatrix} 2 & 1 & 4 \\ -2 & 0 & 3 \\ 3 & 2 & -1 \end{vmatrix}+0$$

$$|A|=\begin{vmatrix} -2 & 0 & 3 \\ 0 & 3 & -2 \\ 3 & 2 & -1 \end{vmatrix}\begin{vmatrix} -2 & 0 \\ 0 & 3 \\ 3 & 2 \end{vmatrix}-1\begin{vmatrix} 2 & 1 & 4 \\ 0 & 3 & -2 \\ 3 & 2 & -1 \end{vmatrix}\begin{vmatrix} 2 & 1 \\ 0 & 3 \\ 3 & 2 \end{vmatrix}-2\begin{vmatrix} 2 & 1 & 4 \\ -2 & 0 & 3 \\ 3 & 2 & -1 \end{vmatrix}\begin{vmatrix} 2 & 1 \\ -2 & 0 \\ 3 & 2 \end{vmatrix}+0$$

$|A|=[(6+0+0)-(27+8+0)]-[(-6-6+0)-(36-8+0)]-2[(0+9-16)-(0+12+2)]$

$|A|=-29+40+42=53$

طريقة إيجاد المحدد الرباعي باستعمال خواص المحددات

١- نختار الصف الثاني

$$|A|=\begin{vmatrix} 2 & 1 & 4 & -1 \\ -2 & 0 & 3 & -1 \\ 0 & 3 & -2 & 2 \\ 3 & 2 & -1 & 0 \end{vmatrix}$$

٢- نضرب العمود الرابع في (3) ونضيفه الى العمود الثالث

$$|A|=\begin{vmatrix} 2 & 1 & 1 & -1 \\ -2 & 0 & 0 & -1 \\ 0 & 3 & 4 & 2 \\ 3 & 2 & -1 & 0 \end{vmatrix}$$

٣- نضرب العمود الرابع في (-٢) ونضيفه الى العمود الاول

$$|A|=\begin{vmatrix} 4 & 1 & 1 & -1 \\ 0 & 0 & 0 & -1 \\ -4 & 3 & 4 & 2 \\ 3 & 2 & -1 & 0 \end{vmatrix}$$

$$|A|=(-1)^{2+4}X-1\begin{vmatrix} 4 & 1 & 1 \\ -4 & 3 & 4 \\ 3 & 2 & -1 \end{vmatrix}\begin{vmatrix} 4 & 1 \\ -4 & 3 \\ 3 & 2 \end{vmatrix}$$

$$|A|=-1[(-12+12-8)-(9+32+4)]=-1[-8-45]=53$$

طريقة حل اخرى

١-نختار العمود الثاني

$$|A| = \begin{vmatrix} 2 & 1 & 4 & -1 \\ -2 & 0 & 3 & -1 \\ 0 & 3 & -2 & 2 \\ 3 & 2 & -1 & 0 \end{vmatrix}$$

٢- نضرب الصف الأول في (3-) ونضيفه الى الصف الثالث

$$|A| = \begin{vmatrix} 2 & 1 & 4 & -1 \\ -2 & 0 & 3 & -1 \\ -6 & 0 & -14 & 5 \\ 3 & 2 & -1 & 0 \end{vmatrix}$$

٣- نضرب الصف الأول في (-٢) ونضيفه الى الصف الرابع

$$|A| = \begin{vmatrix} 2 & 1 & 4 & -1 \\ -2 & 0 & 3 & -1 \\ -6 & 0 & -14 & 5 \\ -1 & 0 & -9 & 2 \end{vmatrix}$$

$$|A| = (-1)^{1+2} X1 \begin{vmatrix} -2 & 3 & -1 \\ -6 & -14 & 5 \\ -1 & -9 & 2 \end{vmatrix} \begin{vmatrix} -2 & 3 \\ -6 & -14 \\ -1 & -9 \end{vmatrix}$$

$$|A| = -[(56-15-54)-(-14+90-36)] = -[-13-40) = 53$$

Ex2:

$$\begin{vmatrix} 4 & 3 & 2 & 2 \\ 0 & 1 & -3 & 3 \\ 0 & -1 & 3 & 3 \\ 0 & 3 & 1 & 1 \end{vmatrix}$$

Since there is only one element different from 0 on column 1, we apply the general formula using this column. The cofactors corresponding to the elements which are 0 don't need to be calculated because the product of them and these elements will be 0.

$$\begin{vmatrix} \not{4} & \not{3} & \not{2} & \not{2} \\ \not{0} & 1 & -3 & 3 \\ \not{0} & -1 & 3 & 3 \\ \not{0} & 3 & 1 & 1 \end{vmatrix}$$

$$= 4(1 \cdot 3 \cdot 1 + (-1) \cdot 1 \cdot 3 + 3 \cdot (-3) \cdot 3 - (3 \cdot 3 \cdot 3 + 3 \cdot 1 \cdot 1 + 1 \cdot (-3) \cdot (-1))) = 4(3 - 3 - 27 - (27 + 3 + 3)) = 4 \cdot (-60) = -240$$

Next week

- **Solving Systems of Linear Equations by matrices**

شكرا للإصغاء

Mathematics

Lecture 8

Cramer's rule

(Solving Systems of Linear Equations)

Dr. Alaa Alasadi.

Cramer's rule is used to solve systems of linear equations. It applies to systems that meet the following conditions:

- The number of equations equals the number of unknowns.
- The determinant of the coefficient matrix is nonzero.
- So, for system of linear equations like these:

$$a_1x + a_1y + a_1z = c_1 \dots . \dots 1$$
$$a_2x + a_2y + a_2z = c_2 \dots\dots\dots 2$$
$$a_3x + a_3y + a_3z = c_{3\dots} \dots\dots\dots 3$$

a is constant
x, y and z are variables

We can sort these equation to three matrices:

يتم صياغة المعادلات الثلاث الى ثلاث مصفوفات

١- مصفوفة الثوابت

٢- مصفوفة المجاهيل

٣- مصفوفة النواتج

X. Y. Z

$$\begin{bmatrix} a_{11} & a_{12} & a_{31} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{x} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

A is the determinant of the coefficient matrix.

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & . & . & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & . & . & a_{2,n} \\ a_{3,1} & a_{3,2} & a_{3,3} & . & . & a_{3,n} \\ \cdots & & & & & \\ a_{m,1} & a_{m,2} & a_{m,3} & . & . & a_{m,n} \end{pmatrix}$$

If the constant terms are 0, the system is homogeneous

The Associated Matrix is Square (m=n)

We calculate the determinant of the associated matrix.

$$A = \begin{vmatrix} a_{1,1} & a_{1,2} & a_{1,3} & . & . & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & . & . & a_{2,n} \\ a_{3,1} & a_{3,2} & a_{3,3} & . & . & a_{3,n} \\ \cdots & & & & & \\ a_{n,1} & a_{n,2} & a_{n,3} & . & . & a_{n,n} \end{vmatrix}$$

$$X = \frac{\begin{vmatrix} b_1 & a_{1,2} & a_{1,3} & . & . & a_{1,n} \\ b_2 & a_{2,2} & a_{2,3} & . & . & a_{2,n} \\ b_3 & a_{3,2} & a_{3,3} & . & . & a_{3,n} \\ \cdots & & & & & \\ b_n & a_{n,2} & a_{n,3} & . & . & a_{n,n} \end{vmatrix}}{\boldsymbol{A}}$$

$$Y = \frac{\begin{vmatrix} a_{1,1} & b_1 & a_{1,3} & . & . & a_{1,n} \\ a_{2,1} & b_2 & a_{2,3} & . & . & a_{2,n} \\ a_{3,1} & b_3 & a_{3,3} & . & . & a_{3,n} \\ \cdots & & & & & \\ a_{n,1} & b_n & a_{n,3} & . & . & a_{n,n} \end{vmatrix}}{\boldsymbol{A}}$$

$$Z = \frac{\begin{vmatrix} a_{1,1} & a_{1,2} & b_1 & . & . & a_{1,n} \\ a_{2,1} & a_{2,2} & b_2 & . & . & a_{2,n} \\ a_{3,1} & a_{3,2} & b_3 & . & . & a_{3,n} \\ \cdots & & & & & \\ a_{n,1} & a_{n,2} & a_n & . & . & a_{n,n} \end{vmatrix}}{\boldsymbol{A}}$$

The Determinant of the Associated Matrix is not 0

Determinants are obtained by replacing the coefficients of the 2nd member (independent terms) in the 1st, 2nd, 3rd and the nth column, respectively.
The system solution is given by the following expressions:

$$X = \frac{A_x}{A}$$

$$Y = \frac{A_y}{A}$$

$$Z = \frac{A_z}{A}$$

Ex1:

$$\begin{cases} 2\cdot x+3\cdot y-5\cdot z=-7 \\ -3\cdot x+2\cdot y+z=-9 \\ 4\cdot x-y+2\cdot z=17 \end{cases}$$

We will sort the tree equations to three matrices:

$$\begin{bmatrix} 2 & 3 & -5 \\ -3 & 2 & 1 \\ 4 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ -9 \\ 17 \end{bmatrix}$$

The matrix associated to the system is

$$A=\left|\begin{array}{ccc|cc} 2 & 3 & -5 & 2 & 3 \\ -3 & 2 & 1 & -3 & 2 \\ 4 & -1 & 2 & 4 & -1 \end{array}\right|$$

We calculate the determinant of the matrix and we get
A=(8+12−15)-(-40-2-18)=65

نستبدل عمود x, y ,z. بعمود النواتج لكل حالة

$A_x=\begin{vmatrix} -7 & 3 & -5 \\ -9 & 2 & 1 \\ 17 & -1 & 2 \end{vmatrix}$,, $A_y=\begin{vmatrix} 2 & -7 & -5 \\ -3 & -9 & 1 \\ 4 & 17 & 2 \end{vmatrix}$ $A_z=\begin{vmatrix} 2 & 3 & -7 \\ -3 & 2 & -9 \\ 4 & -1 & 17 \end{vmatrix}$

We calculate $\Delta_x = \begin{vmatrix} -7 & 3 & -5 \\ -9 & 2 & 1 \\ 17 & -1 & 2 \end{vmatrix} = -28 - 45 + 51 + 170 - 7 + 54 = 195$

We calculate $\Delta_y = \begin{vmatrix} 2 & -7 & -5 \\ -3 & -9 & 1 \\ 4 & 17 & 2 \end{vmatrix} = -36 + 255 - 28 - 180 - 34 - 42 = -65$

We calculate $\Delta_z = \begin{vmatrix} 2 & 3 & -7 \\ -3 & 2 & -9 \\ 4 & -1 & 17 \end{vmatrix} = 68 - 21 - 108 + 56 - 18 + 153 = 130$

The solution of the system is:

$$x = \frac{\Delta_x}{\Delta} = \frac{195}{65} = 3$$

$$y = \frac{\Delta_y}{\Delta} = -\frac{65}{65} = -1$$

$$z = \frac{\Delta_z}{\Delta} = \frac{130}{65} = 2$$

Ex2

$$4x + 5y - 2z = 3 \ldots\ldots\ldots\ldots(1$$
$$-2x + 3y - z = -3 \ldots\ldots\ldots\ldots(2$$
$$-x - 2y + 3z = -5 \ldots \ldots . . (3$$

We will sort the tree equations to three matrices:

$$\begin{bmatrix} 4 & 5 & -2 \\ -2 & 3 & -1 \\ -1 & -2 & 3 \end{bmatrix} \text{x} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ -5 \end{bmatrix}$$

The matrix associated to the system is

$$A = \left|\begin{array}{ccc|cc} 4 & 5 & -2 & 4 & 5 \\ -2 & 3 & -1 & -2 & 3 \\ -1 & -2 & 3 & -1 & -2 \end{array}\right|$$

We calculate the determinant of the matrix and we get:

$A=36-8+5-6-8+30=49$

To calculate A_x ,A_y *and* A_z *we will replace*

$$A_x = \begin{vmatrix} 3 & 5 & -2 \\ -3 & 3 & -1 \\ -5 & -2 & 3 \end{vmatrix} = 27 - 12 + 25 - 30 - 6 + 45 = 49$$

$$A_y = \begin{vmatrix} 4 & 3 & -2 \\ -2 & -3 & -1 \\ -1 & -5 & 3 \end{vmatrix} = -36 - 20 + 3 + 6 - 20 + 18 = -49$$

$$A_z = \begin{vmatrix} 4 & 5 & 3 \\ -2 & 3 & -3 \\ -1 & -2 & -5 \end{vmatrix} = -60 + 12 + 15 + 9 - 24 - 50 = -98$$

The solution of the system is:

$$X = \frac{A_x}{A} = \frac{49}{49} = 1$$

$$Y = \frac{A_y}{A} = \frac{-49}{49} = -1$$

$$Z = \frac{A_z}{A} = \frac{-98}{49} = -2$$

$$\begin{aligned} 2y - 4z &= -1 \\ -2z + 3y - x &= 4 \\ y - 2z + 3x &= 6 \end{aligned}$$

نعيد ترتيب المعادلات

$$\begin{aligned} 2y - 4z &= -1 \\ -x + 3y - 2z &= 4 \\ 3x + y - 2z &= 6 \end{aligned}$$

$$\begin{bmatrix} 0 & 2 & -4 \\ -1 & 3 & -2 \\ 3 & 1 & -2 \end{bmatrix} \text{x} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix}$$

$$A=\begin{vmatrix} 0 & 2 & -4 \\ -1 & 3 & -2 \\ 3 & 1 & -2 \end{vmatrix}\begin{matrix} 0 & 2 \\ -1 & 3 \\ 3 & 1 \end{matrix}\Bigg| = (0-12+4)-(-36+0+4) = -8-(-32) = 24$$

$$Ax=\begin{vmatrix} -1 & 2 & -4 \\ 4 & 3 & -2 \\ 6 & 1 & -2 \end{vmatrix}\begin{matrix} -1 & 2 \\ 4 & 3 \\ 6 & 1 \end{matrix}\Bigg| = (6-24-16)-(-72+2-16) = -34+86 = 52$$

$$X = \frac{A_x}{A} \qquad X = \frac{52}{24}$$

$$Ax=\begin{vmatrix} -1 & 2 & -4 \\ 4 & 3 & -2 \\ 6 & 1 & -2 \end{vmatrix}\begin{matrix} -1 & 2 \\ 4 & 3 \\ 6 & 1 \end{matrix}\Bigg| =(6-24-16)-(-72+2-16)=-34+86=52$$

$$X=\frac{A_x}{A} \qquad X=\frac{52}{24}$$

$$Ay=\begin{vmatrix} 0 & -1 & -4 \\ -1 & 4 & -2 \\ 3 & 6 & -2 \end{vmatrix}\begin{matrix} 0 & -1 \\ -1 & 4 \\ 3 & 6 \end{matrix}\Bigg| =?$$

$$y=\frac{A_y}{A}$$

$$Az=\begin{vmatrix} 0 & 2 & -1 \\ -1 & 3 & 4 \\ 3 & 1 & 6 \end{vmatrix}\begin{matrix} 0 & 2 \\ -1 & 3 \\ 3 & 1 \end{matrix}\Bigg| =?$$

$$z=\frac{A_z}{A}$$

$$\begin{aligned} 2x + 5 - 4z &= -x \\ -2z + 3x - 2y &= 4y \\ 6 - 2z + 3x &= 0 \end{aligned}$$

نعيد ترتيب المعادلات

$$\begin{aligned} 3x - 4z &= -5 \\ 3x - 6y - 2z &= 0 \\ 3x - 2z &= -6 \end{aligned}$$

$$\begin{bmatrix} 3 & 0 & -4 \\ 3 & -6 & -2 \\ 3 & 0 & -2 \end{bmatrix} \text{x} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ -6 \end{bmatrix}$$

$$A = \left|\begin{array}{ccc|cc} 3 & 0 & -4 & 3 & 0 \\ 3 & -6 & -2 & 3 & -6 \\ 3 & 0 & -2 & 3 & 0 \end{array}\right| = (36-0-0)-(72+0-0) = 36-72 = -36$$

لحساب قيمة y نستبدل عمود النواتج بعمود y

$$A_y = \left|\begin{array}{ccc|cc} 3 & -5 & -4 & 3 & -5 \\ 3 & 0 & -2 & 3 & 0 \\ 3 & -6 & -2 & 3 & -6 \end{array}\right| = (0+30+72)-(0+36+30) = 102-66 = 36$$

$$y = \frac{A_y}{A} \qquad y = \frac{36}{-36} = -1$$

Home works: find the values of x,y and z for the three equations

H. W 1

$$2 \cdot x + 3 \cdot y - 5 \cdot z = 0$$
$$-3 \cdot x + 2 \cdot y + z = 0$$
$$4 \cdot x - y + 2 \cdot z = 0$$

H.W2

$$2 \cdot x + 3 \cdot y + 2 \cdot z = 5$$
$$-3 \cdot x + 2 \cdot y - 3 \cdot z = -1$$
$$4 \cdot x - y + 4 \cdot z = 3$$

Mathematics

Lecture 9

Inverse of matrices

Dr. Alaa Alasadi.

Inverse of matrices

To find the inverse of matrices (A^{-1}), we should calculate that for each element of matrix by (adjoint matrix)

$$A^{-1} = \frac{adj.(A)}{|A|}$$

$$a^{-1} = \frac{adj.(a)}{|A|}$$

a^{-1} is the inverse of element

$$adj(a) = (-1^{i+j})|A_{iJ}|$$

$$(-1^{i+j})$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Ex: Fine the inverse of A: $A = \begin{vmatrix} 2 & -1 & 4 \\ 3 & 0 & -2 \\ 5 & -2 & 3 \end{vmatrix}$

$$A = \begin{vmatrix} 2 & -1 & 4 \\ 3 & 0 & -2 \\ 5 & -2 & 3 \end{vmatrix}\begin{matrix} 2 & -1 \\ 3 & 0 \\ 5 & -2 \end{matrix}\Bigg|$$

A=(0+10-24)-(0+8-9)=-14+1=-13

$$adj(a) = (-1^{i+j})|A_{iJ}|$$

1- $adj(a_{11}) = (-1^{1+1})|A_{11}|$

$$adj(a_{11}) = (-1^{1+1})\begin{vmatrix} 0 & -2 \\ -2 & 3 \end{vmatrix}$$

$$adj(a_{11}) = (0 * 3 - (-2 * -2)) = -4$$

2- $adj(a_{12}) = (-1^{1+2})|A_{12}|$

$$adj(a_{12}) = (-1^{1+2})\begin{vmatrix} 3 & -2 \\ 5 & 3 \end{vmatrix}$$

A=-1((9-(-10))=-1(19)=-19

$$adj(a_{12}) = -19$$

$$A = \begin{vmatrix} 2 & -1 & 4 \\ 3 & 0 & -2 \\ 5 & -2 & 3 \end{vmatrix}$$

3- $adj(a_{13}) = (-1^{1+3})|A_{13}|$ $adj(a_{12}) = (-1^{1+3}) \begin{vmatrix} 3 & 0 \\ 5 & -2 \end{vmatrix}$

$adj(a_{13}) = -6$

4- $adj(a_{21}) = (-1^{2+1}) \begin{vmatrix} -1 & 4 \\ -2 & 3 \end{vmatrix}$=-5

5- $adj(a_{22}) = (-1^{2+2}) \begin{vmatrix} 2 & 4 \\ 5 & 3 \end{vmatrix}$=-14

6- $adj(a_{23}) = (-1^{2+3}) \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$=-1

7- $adj(a_{31}) = (-1^{3+1}) \begin{vmatrix} -1 & 4 \\ 0 & -2 \end{vmatrix}$=2

8- $adj(a_{32}) = (-1^{3+2}) \begin{vmatrix} 2 & 4 \\ 3 & -2 \end{vmatrix}$=14

9- $adj(a_{33}) = (-1^{3+3}) \begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix}$=3

$$A^{-1} = \begin{vmatrix} 4/13 & 19/13 & 6/13 \\ 5/13 & 14/13 & 1/13 \\ 2/-13 & 14/-13 & 3/-13 \end{vmatrix}$$

$$A * A^{-1} = I$$

$$\begin{vmatrix} 2 & -1 & 4 \\ 3 & 0 & -2 \\ 5 & -2 & 3 \end{vmatrix} * \begin{vmatrix} 4/13 & 19/13 & 6/13 \\ 5/13 & 14/13 & 1/13 \\ 2/-13 & 14/-13 & 3/-13 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Next week

- **Derivatives, Differentials**

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Mathematics

Lecture 8

Derivatives, Differentials

Dr. Alaa Alasadi.

The Most Important Derivatives - Basic Formulas/Rules

1- $\frac{d}{dx}(c) = 0$ Where c is a constant $y=5,\quad \frac{dy}{dx} = 0$

2- $\frac{d}{dx}(x) = 1$ $y=x,\quad \frac{dy}{dx} = 1$

3- $\frac{d}{dx}(cx) = c\frac{d}{dx}x = c$ $\frac{d}{dx}(5x) = 5$

4- $\frac{d}{dx}(cx^n) = ncx^{n-1}$ $y= x^3,\quad \frac{dy}{dx} = 3x^2$

5- $\frac{d}{dx}(u \pm v \pm w \pm \cdots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \pm \cdots$

Ex1: $y = x^4 + 2x - 4$, … $\frac{dy}{dx} = 4x^3 + 2$

Ex2: $y = 3x2 + \sqrt{7}x + 1$ … $\frac{dy}{dx} = 6x + \sqrt{7}$.

6- $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

7- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v(\frac{du}{dx})-u(\frac{dv}{dx})}{v^2}$

Ex- $y = 8 - \frac{2x^2}{5}$ …. $\frac{dy}{dx} = 0 - \frac{5*(-4x)+2x^2*0}{25} = \frac{20x}{25}$.

$$1.\ \frac{d}{dx}\left(x^{7}\right) = 7x^{7-1} = 7x^{6}$$

$$2.\ \frac{d}{dx}\left(x^{256}\right) = 256x^{256-1} = 256x^{255}$$

$$3.\ \frac{d}{dx}\left(x^{1000,000,000,000}\right) = 1,000,000,000,000x^{1000,000,000,000-1} = 1,000,000,000,000x^{999,999,999,999}$$

$$4.\ \frac{d}{dx}\left(x^{-3}\right) = -3x^{-3-1} = -3x^{-4}$$

$$5.\ \frac{d}{dx}\left(x^{\frac{7}{9}}\right) = \frac{7}{9}x^{\frac{7}{9}-1} = \frac{7}{9}x^{\frac{-2}{9}}$$

$$6.\ \frac{d}{dx}\left(x^{\frac{-1}{2}}\right) = \frac{-1}{2}x^{\frac{-1}{2}-1} = \frac{-1}{2}x^{\frac{-3}{2}}$$

$$7.\ \frac{d}{dx}\left(x^{0.368}\right) = 0.368x^{0.368-1} = 0.368x^{-0.632}$$

$$8.\ \frac{d}{dx}\left(x\right) = \frac{d}{dx}\left(x^{1}\right) = 1x^{1-1} = 1$$

$$9.\ \frac{d}{dx}\left(\sqrt{x}\right) = \frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{\frac{-1}{2}}$$

1. $\frac{d}{dx}\left(3x^7\right) = \frac{d}{dx}\left(3x^7\right) = 3\frac{d}{dx}\left(x^7\right) = (3)7x^6 = 21x^6$

2. $\frac{d}{dx}(-5x) = \frac{d}{dx}(-5x) = -5\frac{d}{dx}(x) = -5(1) = -5$

1. $\frac{d}{dx}\left(3x^7+5\right)=\frac{d}{dx}\left(3x^7\right)+\frac{d}{dx}(5)=3\frac{d}{dx}\left(x^7\right)+\frac{d}{dx}(5)=3\left(7x^6\right)+0=21x^6$

Home work

1- $y = 12x^5 + 3x^4 + 7x^3 + x^2 - 9x + 6$

2- $y = \frac{2x+3}{x+1}$

3- $y = (5x^3 + 6)(x^{0.5} - x)$

Mathematics

Lecture 11

Derivatives, Differentials

(Derivatives of Exponential and Logarithm Functions)

Dr. Alaa Alasadi.

For the natural exponential function:

$8 - f(u) = e^u \quad , \frac{d}{dx}(e^u) = e^u .du$

Ex1-

$y = e^2$

Sol.

$\frac{dy}{dx} = e^2.(0)=0$

Ex2-

$y = e^x$

Sol.

$\frac{dy}{dx} = e^x.$

Ex3-

$y = e^{2x}$

Sol.

$\frac{dy}{dx} = e^{2x}.2$

Ex4-

$y = e^{5-3x}$

sol-

$\frac{dy}{dx} = e^{5-3x}.(-3)$

$y = e^{x^2},,,\frac{dy}{dx} = e^{x^2}.2x$

Find the derffivtave of following function

$y = 1/e^{x^2},,,\frac{dy}{dx} = ((e^{x^2}.0)-1.\ e^{x^2}.2x)/\ (e^{x^2})^2 = 2x/e^{x^2}$

$Y= 2x/e^{x^2},,,\frac{dy}{dx} = [(e^{x^2}.2) - (2x.\ e^{x^2}.2x)]/\ (e^{x^2})^2$

$\frac{dy}{dx} = [2.e^{x^2} - 4x^{2\cdot}\,(e^{x^2})]/(e^{x^2})^2$

the derivative of the natural logarithm function is.

$9 - \; f(u) = \ln u, \qquad \frac{d}{dx}(\ln u) = \frac{1}{u}.du$

Ex1-

$y = \ln x$

Sol.

$\frac{dy}{dx} = \frac{1}{x}$

Ex2-

$y = \ln x^2$

Sol.

$\frac{dy}{dx} = \frac{1}{x^2} .2x$

Ex3-

$y = \ln e^x$

Sol.

$\frac{dy}{dx} = \frac{1}{e^x} .e^x .1 = 1$

$8-\ f(u)=e^{u}$, $\frac{d}{dx}(e^{u})=e^{u}.1.du$

Ln2.7=1

$10-\ f(u)=au,$ $\frac{d}{dx}(au)=a^{u}.\ln a.du$

$9-\ f(u)=\ln u,$ $\frac{d}{dx}(\ln u)=\frac{1}{u.ln2.7}.du$

$11-\ f(u)=\log_{a}u,$ $\frac{d}{dx}(\log_{a}u)=\frac{1}{u.\ln a}.du$

Ex2-

$y=\log_{3}5^{x}$

2.7

Sol.

Y=2.7^{x}

$\frac{dy}{dx}=\frac{1}{5^{x}ln3}.5^{x}.\ln5.1=\ln5/\ln3$

10 – $f(u) = au$, $\frac{d}{dx}(au) = a^u . \ln a . du$

Where *a* is constant

Ex1-

$y = 5^x$

Sol.

$$\frac{dy}{dx} = 5^x . \ln 5$$

Ex2-

$y = 2^{x^2}$

Sol.

$$\frac{dy}{dx} = 2^{x^2} . \ln 2 . 2x$$

Ex3-

$y = 7^{e^x}$

Sol.

$$\frac{dy}{dx} = 7^{e^x} . \ln 7 . e^x$$

Ex4-

$y = 9^{(3x-5)}$

sol-

$$\frac{dy}{dx} = 9^{(3x-5)} . \ln 9 . 3$$

11 – $f(u) = \log_a u$, $\frac{d}{dx}(\log_a u) = \frac{1}{u.\ln a}.du$

Where *a* is constant

$\log_a x = \frac{\ln x}{\ln a}$

Ex1-

$y = \log_2 x$

Sol.

$\frac{dy}{dx} = \frac{1}{x.ln2}.1$

Ex2-

$y = \log_5 x^2$

Sol.

$\frac{dy}{dx} = \frac{1}{x^2.ln5}.2x$

Ex2-

$y = \log_3 e^x$

Sol.

$\frac{dy}{dx} = \frac{1}{e^x ln3} e^x.1 = \frac{1}{ln3}$

$$y = 1/e^{(log_3^{5^x})}$$

$$\frac{dy}{dx} = e^{log_3^{5^x}} . \frac{1}{5^x . ln3} . 5^x . ln5$$

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Derivatives of the trig functions

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Mathematics

Lecture 10

Derivatives, Differentials

(Derivatives of Trigonometric Functions)

Dr. Alaa Alasadi.

12- $\frac{d}{dx}(\sin(x)) = \cos(x)$

Example 1:

$$y = \sin 5x$$

Solution:

$$\frac{dy}{dx} = 5cos5x.$$

Example 2:

$$y = \sin^2\sqrt{x}$$

Solution:

$$\frac{dy}{dx} = 2\sin\sqrt{x}.cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}.$$

Example 3:

$$y = x.\sin(1 + x^2)$$

Solution:

$$\frac{dy}{dx} = x.cos(1 + x^2).(2x) + \sin(1 + x^2)$$

13- $\frac{d}{dx}(\cos(x)) = -\sin(x)$

Example 1: $y = \cos\frac{1}{x}$

Solution:

$$\frac{dy}{dx} = -\sin\frac{1}{x}.\left(\frac{-1}{x^2}\right).$$

$$\frac{dy}{dx} = \frac{1}{x^2}.\sin\frac{1}{x}$$

Example 2:

$$y = \sin^3 x + \cos^3 x$$

Solution:

$$\frac{dy}{dx} = 3\sin^2 x \cdot cosx + 3\cos^2 x \cdot (-sinx)$$

$$= 3\sin^2 xcosx - 3\cos^2 xsinx$$

$$= 3sinx.cosx(sinx - cosx).$$

Example 3:

$$y = 1/cos^n x$$

Solution:

$$= cos^{-n} x$$

$$= -n(cosx)^{-n-1}(-sinx)$$

$$= \frac{nsinx}{cos^{n+1} x}.$$

14- $\frac{d}{dx}(\tan(x)) = \sec^2(x)$

Example 1.

$$y = 2tanx^2$$

Solution.

$$\frac{dy}{dx} = 2sec^2x^2 . 2x$$

$$\frac{dy}{dx} = 4x . sec^2x^2.$$

Example 2.

$$y = tan^2x^3$$

Solution.

$$\frac{dy}{dx} = 2tanx^3 . sec^2x^3 . 3x$$

$$\frac{dy}{dx} = 6x . tanx^3 . sec^2x^3.$$

Example 3.

$$y = tanx + \frac{1}{3}tan^3x$$

Solution.

$$\frac{dy}{dx} = sec^2x + tan^2x \cdot sec^2x$$

$$\frac{dy}{dx}=\sec^2x(1+\tan^2x)$$

$$= \frac{1 + \tan^2x}{cos^2x}.$$

$$1 + \tan^2x = \sec^2x = 1/\cos 2x$$

$$= \frac{1 + \tan^2x}{cos^2x} = \frac{\frac{1}{\cos^2x}}{cos^2x}$$

$$\frac{dy}{dx} = 1/\cos^4x = \sec^4x.$$

15- $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$

Example 1.

$$y = x.cotx$$

Solution.

$$\frac{dy}{dx} = x.(-csc^2x) + cotx$$

$$\frac{dy}{dx} = -x.csc^2x + cotx.$$

Example 2.

$$y = \tan\frac{x}{2} - \cot\frac{x}{2}$$

Solution.

$$\frac{dy}{dx} = \sec^2\frac{x}{2}.\frac{1}{2} + \csc^2\frac{x}{2}.\frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}(\sec^2\frac{x}{2} + \csc^2\frac{x}{2})$$

16- $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$

17- $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$

Example 1.

$$y = \sec(x^2+1)$$

Solution.

$$\frac{dy}{dx} = \sec(x^2+1).\tan(x^2+1).2x$$

$$\frac{dy}{dx} = 2x.\sec(x^2+1).\tan(x^2+1)$$

Example 2.

$$y = \csc^2\left(\frac{2}{x}\right)$$

Solution.

$$\frac{dy}{dx} = 2csc\left(\frac{2}{x}\right).(-\csc\left(\frac{2}{x}\right).\cot\left(\frac{2}{x}\right).\left(\frac{-2}{x^2}\right))$$

$$\frac{dy}{dx} = \frac{4}{x^2}.csc^2\left(\frac{2}{x}\right).\cot\left(\frac{2}{x}\right)$$

Example 3.

$$y = sec^2\,\frac{x}{2} + csc^2\,\frac{x}{2}$$

Solution.

$$\frac{dy}{dx} = 2sec\frac{x}{2}.sec\frac{x}{2}tan\frac{x}{2}.\frac{1}{2} - 2csc\frac{x}{2}.csc\frac{x}{2}.cot\frac{x}{2}.\frac{1}{2}$$

$$\frac{dy}{dx} = sec^2(\frac{x}{2}).tan(\frac{x}{2}) - csc^2(\frac{x}{2}).cot(\frac{x}{2})$$

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Integration of Functions

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Mathematics

Lecture 12

Derivatives, Differentials

(Derivatives of inverse of Trigonometric Functions)

Dr. Alaa Alasadi.

$$(\sin^{-1}u) \neq \frac{1}{\sin u}$$

$$18 - d(\sin^{-1}u) = \frac{du}{\sqrt{1-u^2}}$$

Ex1:

$$y = (x + \sin^{-1}x^2)^2$$

Sol:

$$\frac{dy}{dx} = 2(x + \sin^{-1}x^2).(1 + \frac{2x}{\sqrt{1-x^4}})$$

Ex3:

$$y = \ln(\sin^{-1}(\sec x)).$$

$$\frac{dy}{dx} = \frac{1}{(\sin^{-1}(\sec x))} \cdot \frac{\sec x . \tan x}{\sqrt{1-(\sec x)^2}}$$

Ex2:

$$y = \sin^{-1}e^x$$

Sol:

$$\frac{dy}{dx} = \frac{e^x . 1}{\sqrt{1-e^{2x}}}$$

Ex4:

$$y = e^{\sin^{-1}x^2}$$

$$\frac{dy}{dx} = e^{\sin^{-1}x^2} . \frac{2x}{\sqrt{1-x^4}}$$

$$(\cos^{-1}u) \neq \frac{1}{\cos u}$$

$$19 - d(\cos^{-1}u) = \frac{-du}{\sqrt{1-u^2}}$$

Ex1:

$$y = 3(\cos^{-1}5x)^2$$

$$\frac{dy}{dx} = 6(\cos^{-1}5x).\frac{-5}{\sqrt{1-25x^2}}.$$

Ex2:

$$y = \log_3(\sin 3x.\cos^{-1}(\sin x))$$

$$\frac{dy}{dx} = \frac{1}{(\sin 3x.\cos^{-1}(\sin x).\ln 3)}.(\sin 3x.\frac{-\cos x}{\sqrt{1-\sin^2 x}} + \cos^{-1}(\sin x).\cos 3x.3)$$

$$(tan^{-1}u) \neq \frac{1}{tanu}$$

$$20 - d(tan^{-1}u) = \frac{du}{1+u^2}$$

Ex1:

$$y = \frac{x}{tan^{-1}x^2}$$

$$\frac{dy}{dx} = \frac{(tan^{-1}x^2) - x.\frac{2x}{1+x^4}}{(tan^{-1}x^2)^2}$$

Ex2:

$$y = (tan^{-1}x^2)^2$$

$$\frac{dy}{dx} = 2(tan^{-1}x^2).\frac{2x}{1+x^4}$$

$$(cot^{-1}u) \neq \frac{1}{cotu}$$

$$21 - d(cot^{-1}u) = \frac{-du}{1+u^2}$$

Ex3:

$$y = \sin(cot^{-1}e^x)$$

$$\frac{dy}{dx} = \cos(cot^{-1}e^x).\frac{-e^x.1}{1+e^{2x}}$$

$$(sec^{-1}u) \neq \frac{1}{secu}$$

$$(csc^{-1}u) \neq \frac{1}{cscu}$$

$$22 - d(sec^{-1}u) = \frac{du}{|u|\sqrt{u^2 - 1}}$$

$$23 - d(csc^{-1}u) = \frac{-du}{|u|\sqrt{u^2 - 1}}$$

Ex1:

$$y = tanx.\cos(sec^{-1}x)$$

$$\frac{dy}{dx} = tanx.(-\sin(sec^{-1}x).\frac{1}{|x|\sqrt{x^2-1}}) + cos(sec^{-1}x).sec^2x$$

Ex2:

$$y = e^{csc^{-1}x}$$

$$\frac{dy}{dx} = e^{csc^{-1}x}.\frac{-1}{x\sqrt{x^2-1}}$$

Ex3:

$$y = log_2\{\sec(csc^{-1}x^2)\}.$$

$$\frac{dy}{dx} = \frac{\sec(csc^{-1}x^2).\ \tan(csc^{-1}x^2)}{\sec(csc^{-1}x^2).ln2} \cdot \frac{-2x}{x^2\sqrt{x^4 - 1}}$$

Next week

Partial derivetave

شكرا للإصغاء

Mathematics

Lecture 13

Implicit Differentiation

Dr. Alaa Alasadi.

If a function is described by the equation $y = f(x)$ where the variable y is on the left side, and the right side depends only on the independent variable x, then the function is said to be given explicitly. For example, the following functions are defined explicitly:

$$y = sinx, y = x2 + 2x + 5, y = lncosx.$$

In many problems, however, the function can be defined in implicit form, that is by the equation

$$F(x, y) = 0$$

Of course, any explicit function can be written in an implicit form. So the above functions can be represented as

$$y - sinx = 0, y - x2 - 2x - 5 = 0, y - lncosx = 0.$$

Example 1

Find the derivative of the function given by the equation

$y^2 = 2px.$ where p is a parameter.

sol

$$(y^2)' = (2px)', \Rightarrow 2yy' = 2y\frac{dy}{dx} = 2p, \Rightarrow y' = p/y, where\ y \neq 0.$$

Example 2.

Differentiate implicitly the function y(x) given by the equation

$$y = \cos(x + y).$$

$$\frac{dy}{dx} = \frac{d}{dx}cos(x + y), \Rightarrow \frac{dy}{dx} = -\sin(x + y) \cdot \left(1 + \frac{dy}{dx}\right),$$

$$\frac{dy}{dx} = -\sin(x + y) - \frac{dy}{dx}\sin(x + y), \Rightarrow \frac{dy}{dx}(1 + \sin(x + y)) = -\sin(x + y),$$

which results in

$$\frac{dy}{dx} = -\frac{\sin(x + y)}{1 + \sin(x + y)}$$

Example 3.

Calculate the derivative

$$x = y - 2siny.$$

Sol.

$$1 = \frac{dy}{dx} - (2siny)', \Rightarrow 1 = \frac{dy}{dx} - 2cosy \cdot \frac{dy}{dx}, \Rightarrow \frac{dy}{dx} = \frac{1}{1 - 2cosy}$$

Next week

- **Integration**

شكرا للإصغاء