وزارة التعليم العالي والبحث العلمي
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الحقيبة التعليمية

لاجزاء المكائن

المرحلة الثانية

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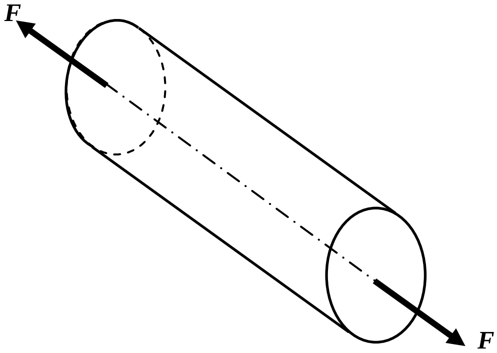
Stresses:

The word stress is used to indicate **force** per unit **area**.

$$\sigma = \frac{F}{A}$$



σ : **Stress (N/m^2)**

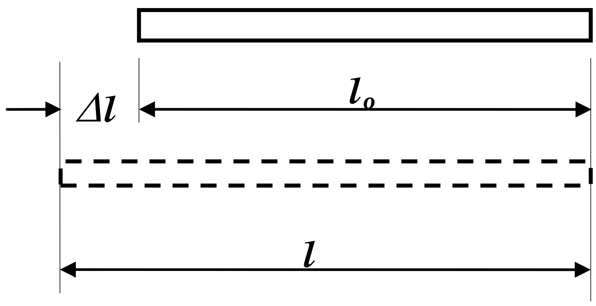
F : **Force (N)**

A : **Area (m^2)**

Strain:

A strain is measured of the deformation of the body.

$$\varepsilon = \frac{l - l_o}{l_o} = \frac{\Delta l}{l_o}$$



ε : **strain**

Stress – Strain Diagram:

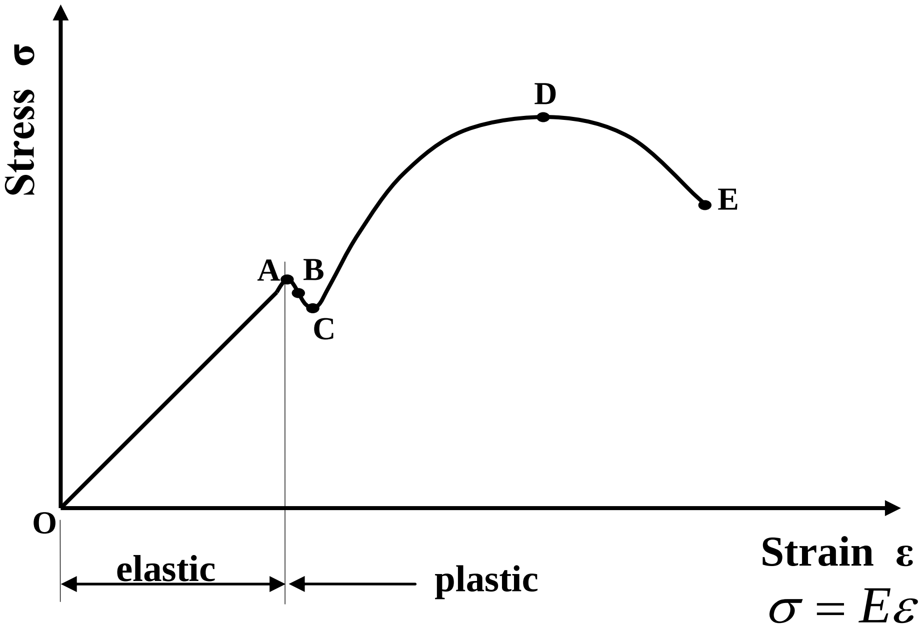
This curve is done by a tensile test to a specimen with a determined dimension, with increasing tensile force gradually and measuring the length of the specimen until fracture.

OA : proportional limit

$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

E : young modulus
Point B : Elastic limit
Point C : yield point
At this point the deformation stay after removing applied force



$$\sigma = E\varepsilon$$

Point D: ultimate tensile stress
Point E: fracture
Beyond point D the external force decrease because of decreasing in cross section area (neck)

$$\sigma = E\varepsilon$$

$$E = \frac{\sigma}{\varepsilon} = \frac{\frac{F}{A}}{\frac{\Delta\ell}{\ell_{\circ}}}$$

$$\boxed{\Delta\ell = \frac{F * \ell_{\circ}}{E * A} = \delta}$$

Safety Factor: S.F

It is a relation between critical stress (yield, ultimate) and allowable stress (calculated)

$$S.F = \frac{\sigma_{critical}}{\sigma_{allowable}}$$

For ductile material (steel, bronze, brass ……) we use yield stress

$$S.F = \frac{\sigma_y}{\sigma_{all}} = \frac{\sigma_y}{\sigma_{cal}}$$

For brittle material (cast iron) we use ultimate stress

$$S.F = \frac{\sigma_{ult}}{\sigma_{all}} = \frac{\sigma_{ult}}{\sigma_{cal}}$$

Factor of safety depend upon the following

1- Degree of economy.
2- Value of strength, yield, and ultimate endurance limits.
3- Load condition (static, dynamic, shock).
4- Degree of accuracy.
5- Importance of machine part.
6- Degree of safety to human life.

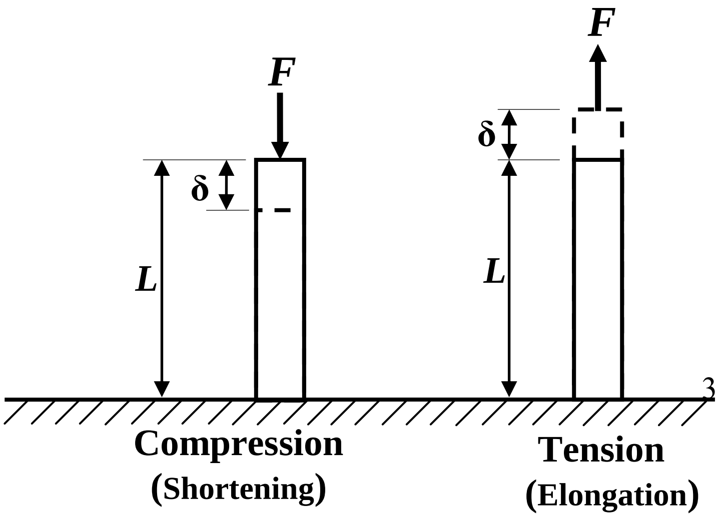
Types of stresses:

A) Tensile and Compressive stress: σ_t, σ_c

Under the effect of this stress, the member will have total elogation or shortening

$$\sigma = \frac{F}{A} \quad \text{N/m}^2$$

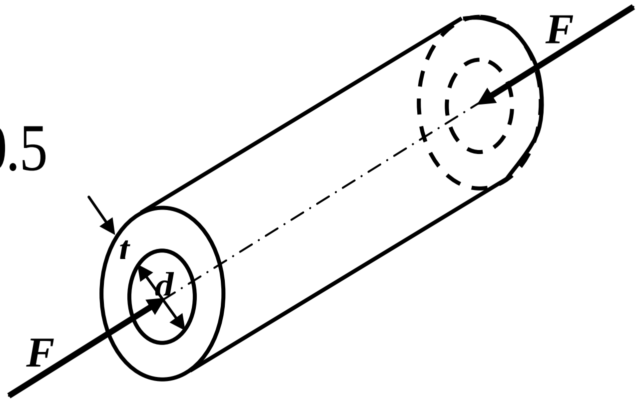
$F \perp A$



Example (1):

Hollow cylinder with length (L =50 cm), thickness (t =0.25 cm) effected by compressive force (F = 2.5 KN) if the compression stress of material (σ_c =70 MN/m^2), modulus of elasticity (E = 200 GN/m^2). Determine the shortening in the length of cylinder.

Solution :



$$D = d + 2*t = d + 0.5 \quad \text{cm}$$

$$A = \frac{\pi}{4}\left[D^2 - d^2\right]$$

$$A = \frac{\pi}{4}\left[(d + 0.5)^2 - d^2\right]$$

$$A = \frac{\pi}{4}\left[d^2 + d + 0.25 - d^2\right] = \frac{\pi}{4}(d + 0.25) \quad \text{cm}^2$$

$$A = \frac{\pi}{4}(d + 0.25)*10^{-4} \quad \text{m}^2$$

$$F = 2.5*10^3 \quad \text{N}$$

$$\sigma_C = 70*10^6 \quad \text{N/m}^2$$

$$E = 200*10^9 \quad \text{N/m}^2$$

$$\sigma_c = \frac{F}{A} \qquad 70*10^6 = \frac{2.5*10^3}{\frac{\pi}{4}(d + 0.25)*10^{-4}}$$

$$d = 0.2047 \quad \text{m}$$

$$A = \frac{\pi}{4}(0.2047 + 0.25)*10^{-4} = 0.3571*10^{-4} \quad \text{m}^2$$

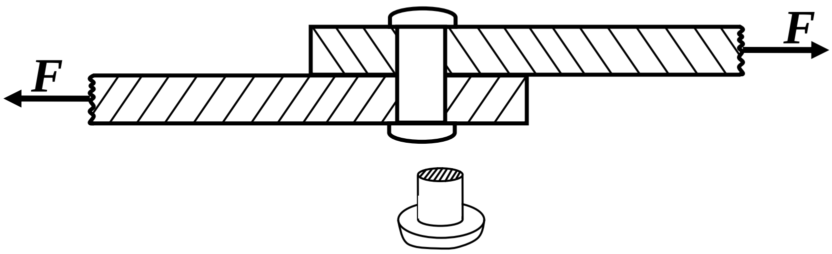
$$\delta = \frac{F * \ell}{E * A} = \frac{2.5 * 10^3 * 0.5}{200 * 10^9 * 0.3571 * 10^{-4}}$$

$$\delta = 0.0002 \quad \textbf{m}$$

$$\delta = 0.2 \quad \textbf{mm}$$

B) Shear stress: τ

If we have two forces effect on the body equal in magnitude and opposite in direction causing relative sliding or slipping of adjacent positions of the body.



$$\tau = \frac{F}{A} \qquad \textbf{(N/m}^2\textbf{)}$$

F ∥ A

τ : **Shear Stress (N/m²)**

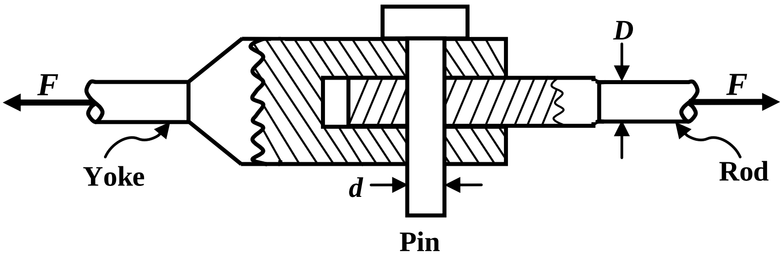
Example (2):

Determine the diameter of the rod (D) and the diameter of the pin (d) for the knuckle joint. If the transmitted force from one side to another through the pin is (80 KN) tension stress for the rod material is (σ_t = 100 MN/m^2) and the shear stress for the pin material is (τ = 80 MN/m^2).

Solution:

For the Rod

$$\sigma_t = \frac{F}{A}$$



$$\sigma_t = 100 * 10^6 \quad \mathbf{(N/m^2)}$$

$$F = 80 * 10^3 \quad \text{N}$$

$$A = \frac{\pi}{4} D^2$$

$$100 * 10^6 = \frac{80 * 10^3}{\frac{\pi}{4} D^2} \Longrightarrow D = 0.0319 \quad \mathbf{m}$$

$$D = 31.9 \quad \mathbf{mm}$$

For the Pin

$$\tau = \frac{F}{2A}$$

$$\tau = 80 * 10^6 \quad \mathbf{(N/m^2)}$$

$$A = \frac{\pi}{4} d^2$$

$$80 * 10^6 = \frac{80 * 10^3}{2 * \frac{\pi}{4} d^2} \Longrightarrow d = 0.0252 \quad \mathbf{m}$$

$$d = 25.2 \quad \mathbf{mm}$$

C) Bearing stress: $\sigma_{bearing}$

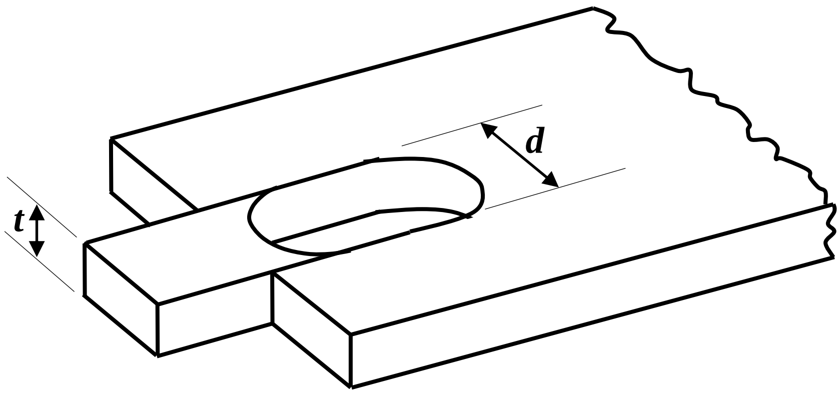
When one object presses against another is referred as bearing stress or (crushing stress)

$$\sigma_{bearing} = \frac{F}{A_{proj}} \qquad \textbf{(N/m}^2\textbf{)}$$

F $\perp$ Aproj

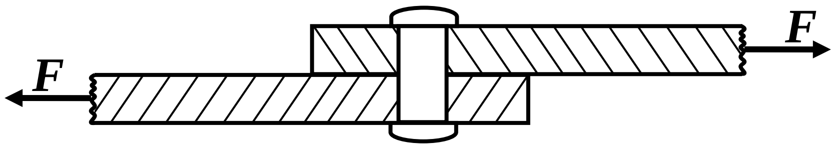
$\sigma_{Bearing}$: **Bearing Stress (N/m^2)**

$$A_{Proj} = t * d$$



Example (3):

Two plates is riveted by lap joint as shown the thickness of plate is ***(16 mm)*** and the diameter of rivet is ***(2.5 cm)*** find the crushing stress when ***(4800 N)*** is applied , and find the shear stress for rivet.



Solution:

$$t = 16mm = 0.016m$$

$$d = 2.5cm = 0.025m$$

Bearing stress

$$\sigma_{bearing} = \frac{F}{A_{proj}} = \frac{F}{t * d} = \frac{4800}{0.016 * 0.025} = 12000000 \frac{N}{m^2}$$

$$\sigma_{bearing} = 12 \frac{MN}{m^2}$$

<u>Shear stress</u>

$$\tau = \frac{F}{A} = \frac{4800}{\frac{\pi}{4}(0.025)^2} = 9778480 \ \frac{N}{m^2}$$

$$\tau = 9.78 \frac{MN}{m^2}$$

Joint:

Some machine parts due to the purpose of holding, adjustment inspection, repair and replacement must be constructed to be read for conection or disconection.

Types of joint:

(1) *permanent joint*

A - Rivet joint
B - Welding joint
C - Pressing joint

(2) *non permanent joint*

A - Screw and bolt joint
B - Keys
C - Shaft coupling
D - Cutter pin joint

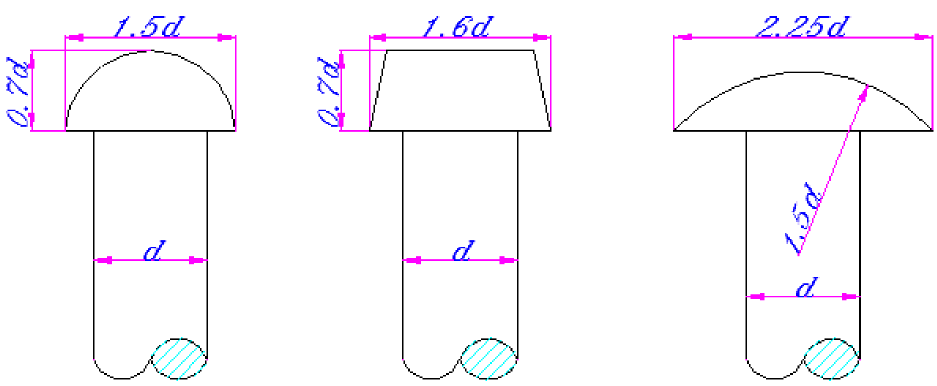
Rivet joints:

It is a permanent fastenings used for various engineering structures such as boilers, bridges, cranes, ships, cars.

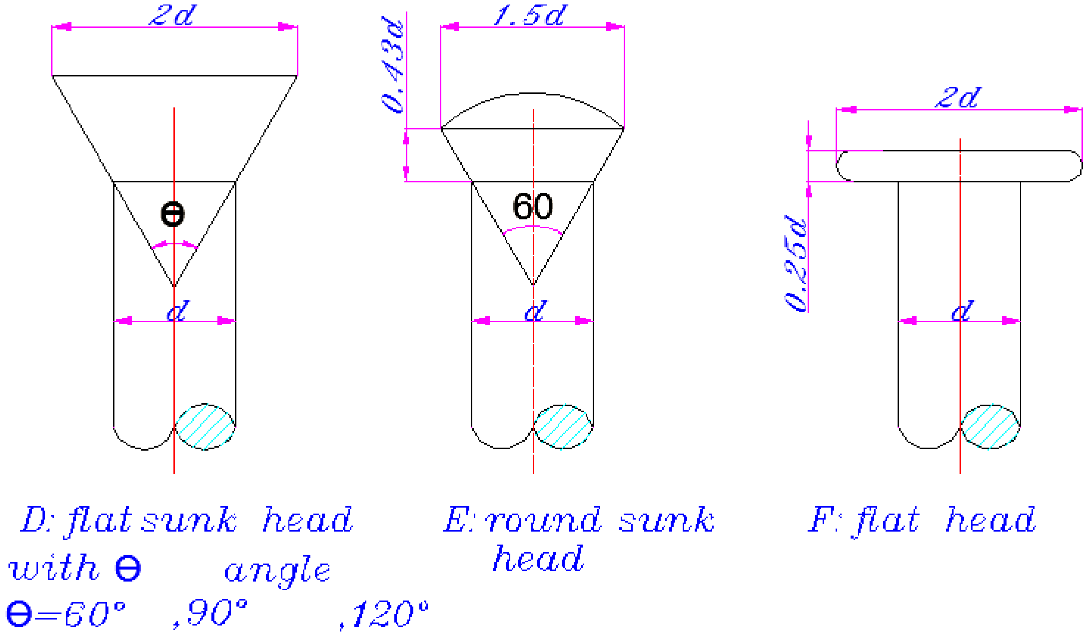
Material: *the rivet are made from*

A- mild steel used for ships, car bodies
B- Aluminium and copper used for airplane bodies

Types of rivet heads and standard dimensions:

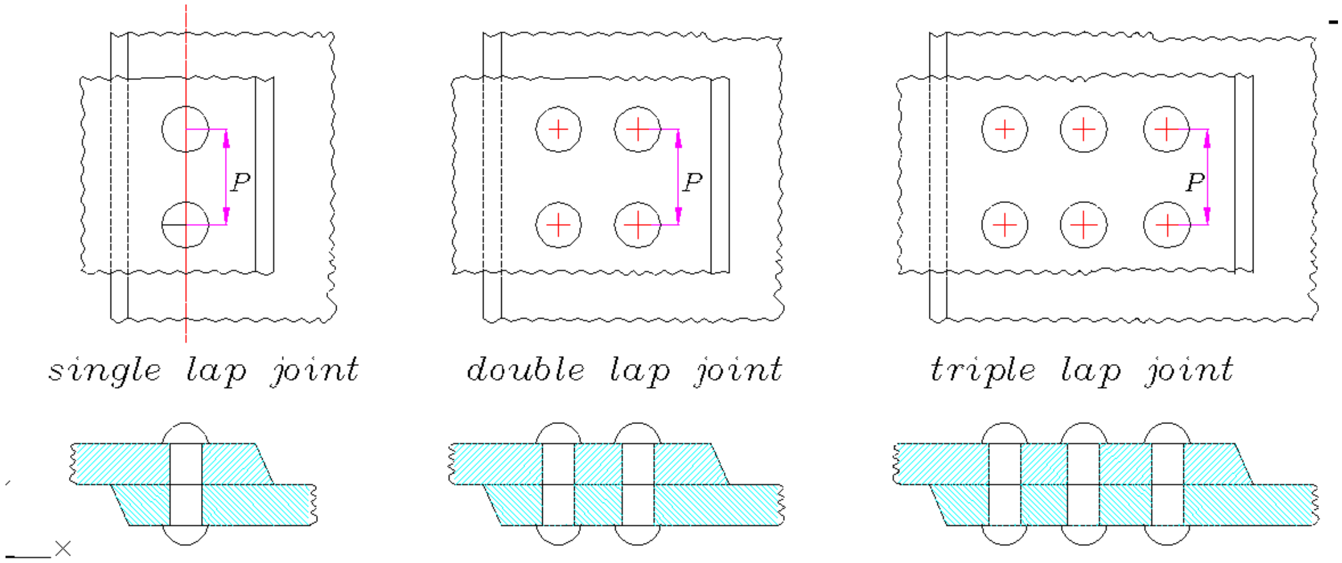


A: snap head B: pan head C: curve head or mushroom head



Types of riveted joints: *there are two types*

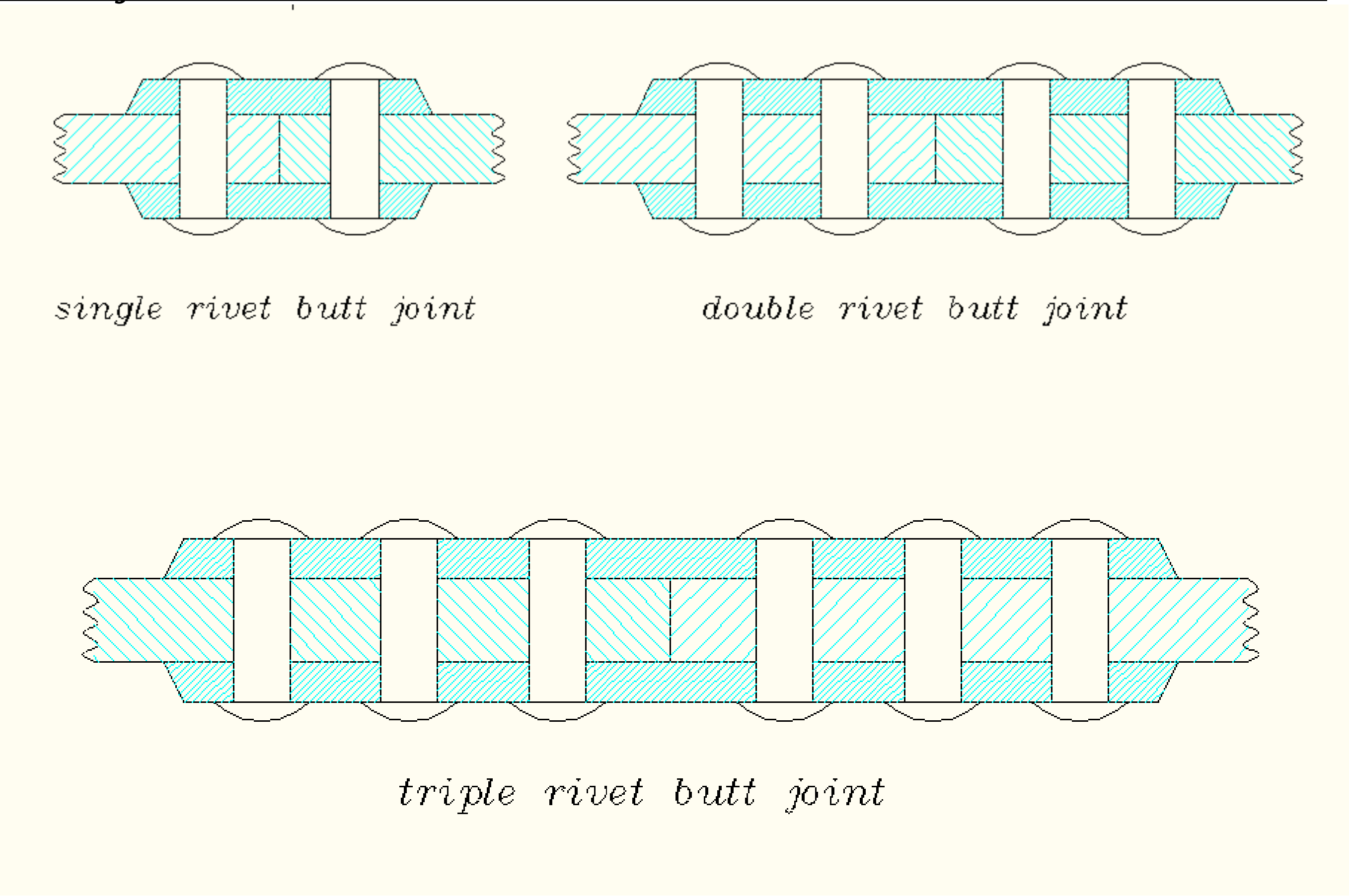
1- Lap joints



2- ***Butt joints:*** *in butt riveting the plates are kept in alignment and a butt strap or cover plate is placed over the joint and riveted to each plate.*

Types of Butt joints:

1- *single rivet butt joint*
2- *double rivet butt joint*
3- *triple rivet butt joint*



Design of rivet joints:

The rivet joint is subjected to a tension force and may be design on the base of following stresses

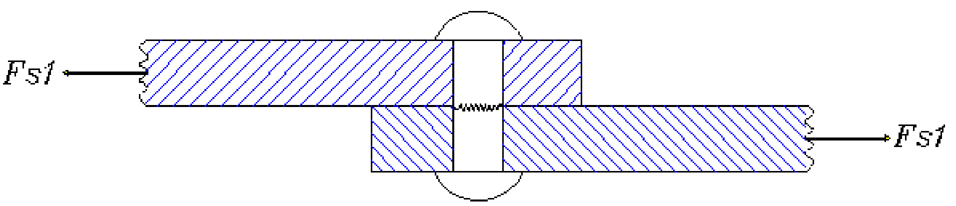
1- Shear stress for rivet:

A- Lap joint:

$$\tau = \frac{F_{S1}}{n \cdot A}$$

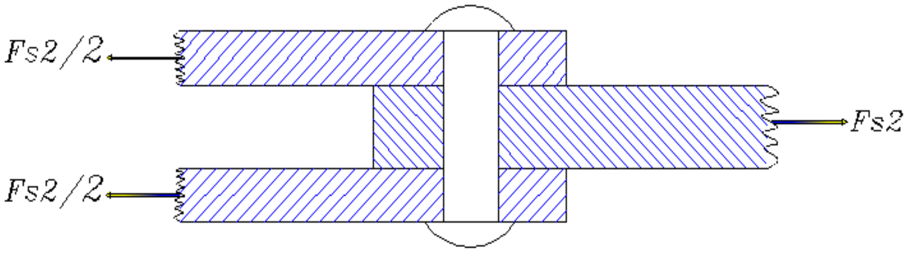
$$F_{s1} = n \cdot A \cdot \tau$$

$$\boxed{F_{s1} = n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau}$$



B- Butt joint:

$$\tau = \frac{F_{S2}}{2 \cdot n \cdot A}$$



$$F_{s2} = 2 \cdot n \cdot A \cdot \tau$$

$$F_{s2} = 2 \cdot n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau$$

Where:
F_{s1} ,F_{s2} : shear force *(N)*
d: diameter of rivet
τ : shear stress ***(N/m²)***
n : number of rows

Note:

1- In practice we take $F_{S2} = 1.875 * F_{S1}$

2- in case of boiler joints $F_{S2} = 1.75 * F_{S1}$

3- in case of n- row of rivets

$$F_{s1} = n \cdot A \cdot \tau$$

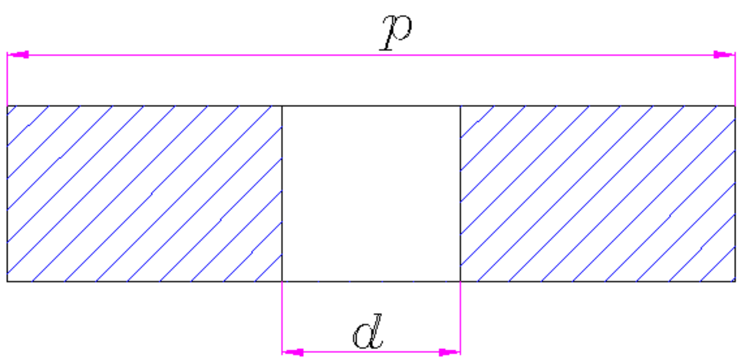
$$F_{s2} = 2 \cdot n \cdot A \cdot \tau$$

Where: *n* : number of rows

2- Tension stress for plate:

$$\sigma_t = \frac{F_t}{A_t}$$

$$F_t = \sigma_t \cdot A_t$$

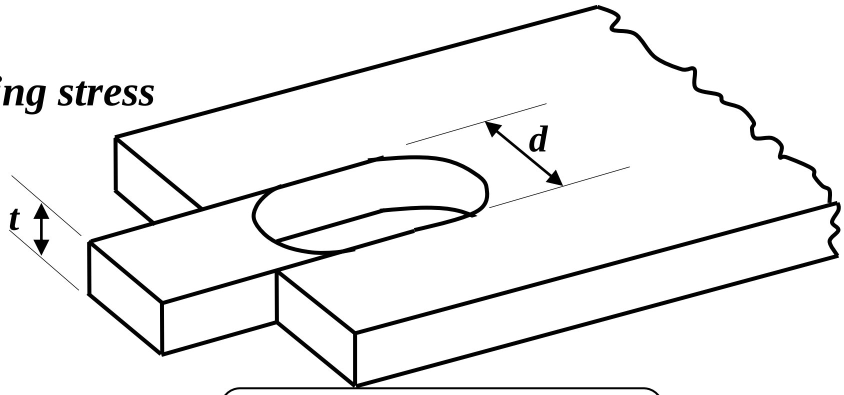


$$F_t = \sigma_t \cdot (P - d) \cdot t$$

Where:
F_t: tensile force *(N)*
p: pitch
σ_t: tensile stress ***(N/m²)***

Crushing stress of plate:

The plate is subjected to crushing stress due to tensile force



$$\sigma_{cr} = \frac{F_{cr}}{n \cdot A_{cr}}$$

$$F_{cr} = \sigma_{cr} \cdot n \cdot A_{cr} \quad \Rightarrow \quad F_{cr} = \sigma_{cr} \cdot n \cdot t \cdot d$$

Where:
F_{cr}: crushing force *(N)*
σ_{cr}: crushing stress ***(N/m²)***

Efficiency of rivet joints:

It is the ability to bear shear, tension and crushing forces. It may be estimated by calculation the cross section area between rivets hole centres as follow

$$A = p \cdot t$$

$$F = \sigma_t \cdot A$$

$$F = \sigma_t \cdot p \cdot t$$

1 ***Shearing efficiency***

A Lap joint

$$\eta_s = \frac{F_{s1}}{F} \times 100\%$$

$$\eta_s = \frac{n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau}{p \cdot t \cdot \sigma_t} \times 100\%$$

(B) Butt joint

$$\eta_s = \frac{F_{s2}}{F} \times 100\%$$

$$\eta_s = \frac{2 \cdot n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau}{p \cdot t \cdot \sigma_t} \times 100\%$$

(2) Tension efficiency

$$\eta_t = \frac{F_t}{F} \times 100\% = \frac{(p-d) \cdot t \cdot \sigma_t}{p \cdot t \cdot \sigma_t} \times 100\%$$

$$\eta_t = \frac{(p-d)}{p} \times 100\%$$

(3) crushing efficiency

$$\eta_{cr} = \frac{F_{cr}}{F} \times 100\% = \frac{n \cdot t \cdot d \cdot \sigma_{cr}}{p \cdot t \cdot \sigma_t} \times 100\%$$

$$\eta_{cr} = \frac{n \cdot d \cdot \sigma_{cr}}{p \cdot \sigma_t} \times 100\%$$

Practical equation for designing the rivet joints:

1- Diameter of rivet depend on the thickness of plate

A: when $t \geq 8mm$

$$d = 6\sqrt{t} \quad mm$$

B: when $t < 8mm$

Lap joint

$$F_{s1} = F_{cr} \Longrightarrow n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = n \cdot t \cdot d \cdot \sigma_{cr} \Longrightarrow d = \frac{4 \cdot t \cdot \sigma_{cr}}{\pi \cdot \tau}$$

Butt joint

$$F_{s2} = F_{cr} \Longrightarrow 2 \cdot n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = n \cdot t \cdot d \cdot \sigma_{cr} \Longrightarrow d = \frac{2 \cdot t \cdot \sigma_{cr}}{\pi \cdot \tau}$$

2- Calculation of pitch

Lap joint

$$F_{s1} = F_t \Longrightarrow n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = (p - d) \cdot t \cdot \sigma_t \qquad p =$$

Butt joint

$$F_{s2} = F_t \Longrightarrow 2 \cdot n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = (p - d) \cdot t \cdot \sigma_t \qquad p =$$

3- The distance between the centre of rivet and the edge of plate must be more than 1.5d

4- Calculation of the thickness of cover plate in case of butt joint

A: in case of one cover plate

$$t_1 = 1.25 \cdot t$$

B: in case of two cover plate

$$t_1 = 0.6t \rightarrow 0.8t$$

Where:

t_1*: thickness of cover plate*

t *: thickness of plate*

Example (1)

*Two **(10 mm)** thick plates are to be jointed by single riveted lap joint. If the tension stress of plate materials is **(σ_t = 100 MN/m²)**, Shear stress of rivet material is **(τ = 70 MN/m²)**. Determine:*

1. *Diameter of Rivet*
2. *Rivet Pitch*
3. *Shearing Efficiency*

Solution :

(1) *because* $t \geq 8mm$

$$d = 6\sqrt{t} = 6\sqrt{10} = 18.97mm$$

(2) *the pitch take form*

$$F_{s1} = F_t \Longrightarrow n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = (p-d) \cdot t \cdot \sigma_t$$

$$1 \times \frac{\pi}{4} \times \left(\frac{18.97}{1000}\right)^2 \times 70 \times 10^6 = \left(p - \frac{18.97}{1000}\right) \times \frac{10}{1000} \times 100 \times 10^6$$

$$p = 0.03877m = 38.77mm$$

(3) *efficiency of shearing*

$$\eta_s = \frac{F_{S1}}{F} \times 100\% = \frac{n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau}{p \cdot t \cdot \sigma_t} \times 100\%$$

$$\eta_s = \frac{1 \times \frac{\pi}{4} \times \left(\frac{18.97}{1000}\right)^2 \times 70 \times 10^6}{\left(\frac{38.77}{1000}\right) \times \left(\frac{10}{1000}\right) \times 100 \times 10^6} \times 100\% = 51\%$$

Example (2)

*A double riveted lap joint is to be made between (**5 mm)** plates. If the safe working stresses are [**(σ_{cr}=110 MN/m^2), (σ_t=70 MN/m^2), (τ =55 MN/m^2)**].calculate the rivet diameter, rivet pitch and state how the joint will fail?*

Solution :

(1) *because* $t < 8mm$

$$F_{s1} = F_{cr}$$

$$n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = n \cdot t \cdot d \cdot \sigma_{cr}$$

$$2 \times \frac{\pi}{4} \times d^2 \times 55 \times 10^6 = 2 \times 0.005 \times d \times 110 \times 10^6$$

$$d = 0.0127m = 12.7mm$$

(2) *the pitch take form*

$$F_{s1} = F_t \Longrightarrow n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = (p-d) \cdot t \cdot \sigma_t$$

$$2 \times \frac{\pi}{4} \times (0.0127)^2 \times 55 \times 10^6 = (p - 0.0127) \times 0.005 \times 70 \times 10^6$$

$$p = 0.05275m = 52.75mm$$

(3) *for fail state we must determine efficiency for all cases*

$$\eta_s = \frac{F_{S1}}{F} \times 100\% = \frac{n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau}{p \cdot t \cdot \sigma_t} \times 100\%$$

$$\eta_s = \frac{2 \times \frac{\pi}{4} \times (0.0127)^2 \times 55 \times 10^6}{(0.05275) \times (0.005) \times 70 \times 10^6} \times 100\% = 75.86\%$$

$$\eta_t = \frac{F_t}{F} \times 100\% = \frac{(p-d) \cdot t \cdot \sigma_t}{p \cdot t \cdot \sigma_t} \times 100\% = \frac{(p-d)}{p} \times 100\%$$

$$\eta_t = \frac{(0.05275 - 0.0127)}{0.05275} \times 100\% = 75.92\%$$

$$\eta_{cr} = \frac{F_{cr}}{F} \times 100\% = \frac{n \cdot t \cdot d \cdot \sigma_{cr}}{p \cdot t \cdot \sigma_t} \times 100\% = \frac{n \cdot d \cdot \sigma_{cr}}{p \cdot \sigma_t} \times 100\%$$

$$\eta_{cr} = \frac{2 \times 0.0127 \times 110 \times 10^6}{0.05275 \times 70 \times 10^6} \times 100\% = 75.67\%$$

The plate will fail in crushing

Example (3)

*Two **(15 mm)** thick plates are to be jointed by triple riveted double cover strap butt joint. If the Shear stress of rivet material is **(τ =61.7 MN/m^2)** and tension stress for plate materials is **(σ_t=82.4 MN/m^2)**. Determine:*

1. Diameter of Rivet.
2. Rivet Pitch.
*3. Cover strap thickness **(t_1)**.*
4. Shearing efficiency of Riveted.

SOLUTION :

(1) *because* $t \geq 8mm$

$$d = 6\sqrt{t} = 6\sqrt{15} = 23.2mm$$

(2) *the pitch take form*

$F_{S2} = F_t \Longrightarrow 2 \cdot n \cdot \frac{\pi}{4} \cdot d^2 \cdot \tau = (p - d) \cdot t \cdot \sigma_t$

$$2\times3\times\frac{\pi}{4}\times(0.0232)^2\times61.7\times10^6=(p-0.0232)\times\frac{15}{1000}\times82.4\times10^6$$

$$p=0.1498m=149.8mm$$

(3) $t_1=0.6t\rightarrow0.8t$

$$t_1=0.7\times t=0.7\times15=10.5mm$$

(4) *efficiency of shearing*

$$\eta_s=\frac{F_{S2}}{F}\times100\%=\frac{2\cdot n\cdot\frac{\pi}{4}\cdot d^2\cdot\tau}{p\cdot t\cdot\sigma_t}\times100\%$$

$$\eta_s=\frac{2\times3\times\frac{\pi}{4}\times(0.0232)^2\times61.7\times10^6}{(0.1498)\times(0.015)\times82.4\times10^6}\times100\%=84.5\%$$

Welding Joint:

It is one of the permanent joint; it was obtained by heating two pieces until fused together or indirectly by using weld metal which is deposited in corner between two surfaces.

Types of welding processes:

1 Forge welding

There are two types of forge welding

A - **Manual forges welding**

In manual forge welding, the two parts are heated to plastic state and the pressure is applied by hand hammer.

B - **Machine forges welding**

The two parts are heated to plastic state and the external pressure is applied by press machine.

2 Electric resistance welding :(E.R.W)

The two parts are pressed together and current is passed from one part to other until the metal is heated to fusion temperature at the joint.

The generated heat (H) during E.R.W is

$$H = K \cdot I^2 \cdot R \cdot t$$

Where:

***K**: constant*

***R**: electric resistance (Ω)*

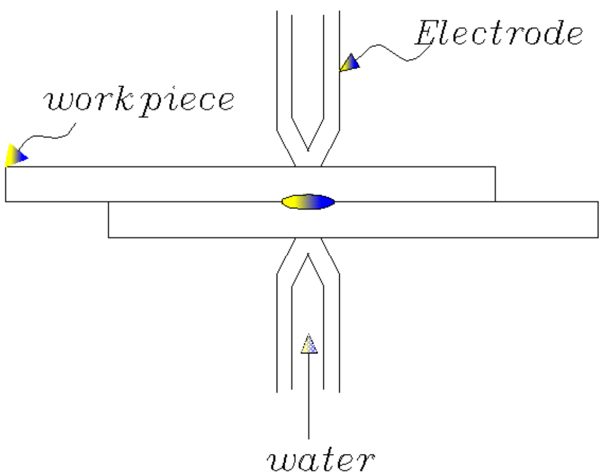
***I**: electric current (A)*

***t**: time (sec)*

There are three types of electric resistance welding

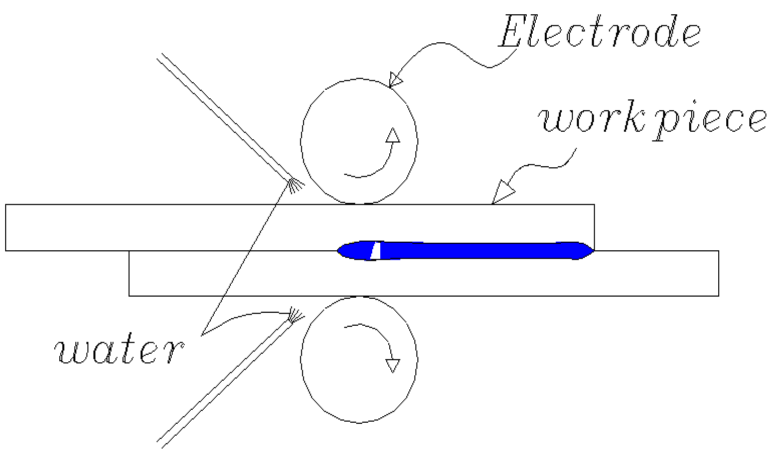
A – **spot welding**

In which we use two-point electrodes on each sides of the cover lapped plates with Appling pressure. This type is very cheap.



B – **seam welding**

In seam welding we use rollers instead of point's electrodes. The plates are to be joint are pulled between the rollers in order to get a uniform continuous strip of welded surface.



C – **flash welding**

This type is depending on air resistance to the passing of electric current from one part to another until the metal is heated to fusion temperature.

3 Fusion (melting) welding:

It is a process of jointing two pieces in a molten state without application of mechanical pressure by heating the joint or member to temperature below the critical T-r of metal.

There types are

A – **Gas welding**

It uses oxy-hydrogen or oxy-acetylene burnt in welding torch. The edges of the work pieces are melting which on cooling results in a strong joint.

B – **Electric arc welding**

The welding temperature is developed by electric arc which is struck between work pieces and electrode which is held by operator or guided automatically.

C – **Thermit welding**

A mixture of iron oxide and aluminium called thermit is ignited and the iron oxide is reduced to molten metal. The advantage of this type is that all parts of the weld section are molten at the same time and the cooling of the weld is uniform.

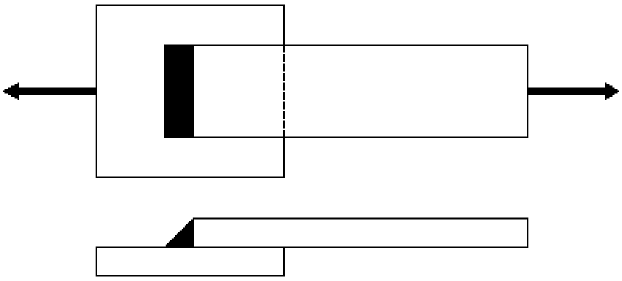
Types of welding joints:

There are two types

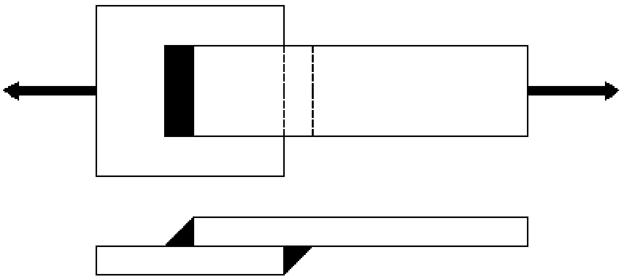
1 Lap welds joints(fillet weld):

The weld metal is deposited in the corner between the two surfaces.
The types are

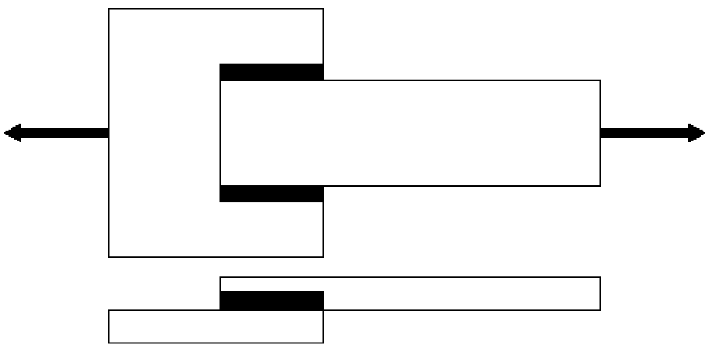
A – **Single transverse fillet**



B – **Double transverse fillet**



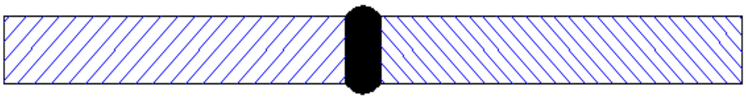
C – **Parallel fillet joint**



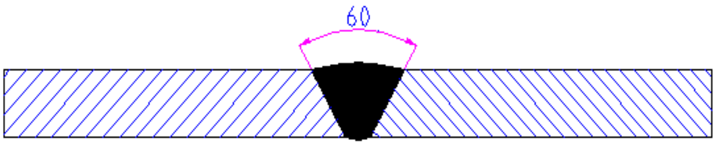
② Butt welds joints:

It is obtained by putting the edges of the two pieces together, there types are:

A – **Square Butt weld**

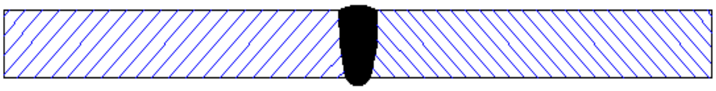


B – **Single V Butt weld**



C – **Double V Butt weld**

D – **U Butt weld**

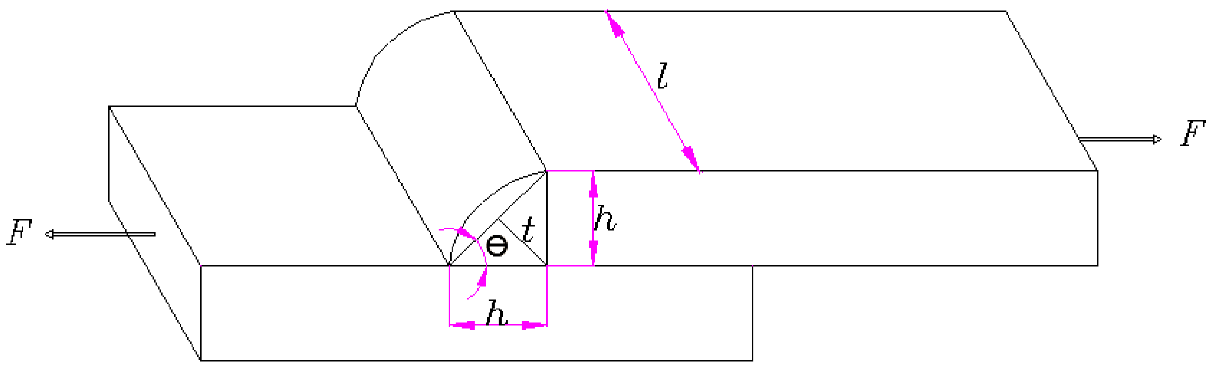


Strength of welds:

① strength of lap welds joints

A- strength of transverse fillet weld

I. For single transverse fillet weld



$$\sin\theta = \frac{t}{h}$$

$$t = h \cdot \sin\theta = h \cdot \sin 45$$

$$\sigma_t = \frac{F}{A_w}$$

Where: $A_w = t \cdot l$

$$\sigma_t = \frac{F}{l \cdot h \cdot \sin 45}$$

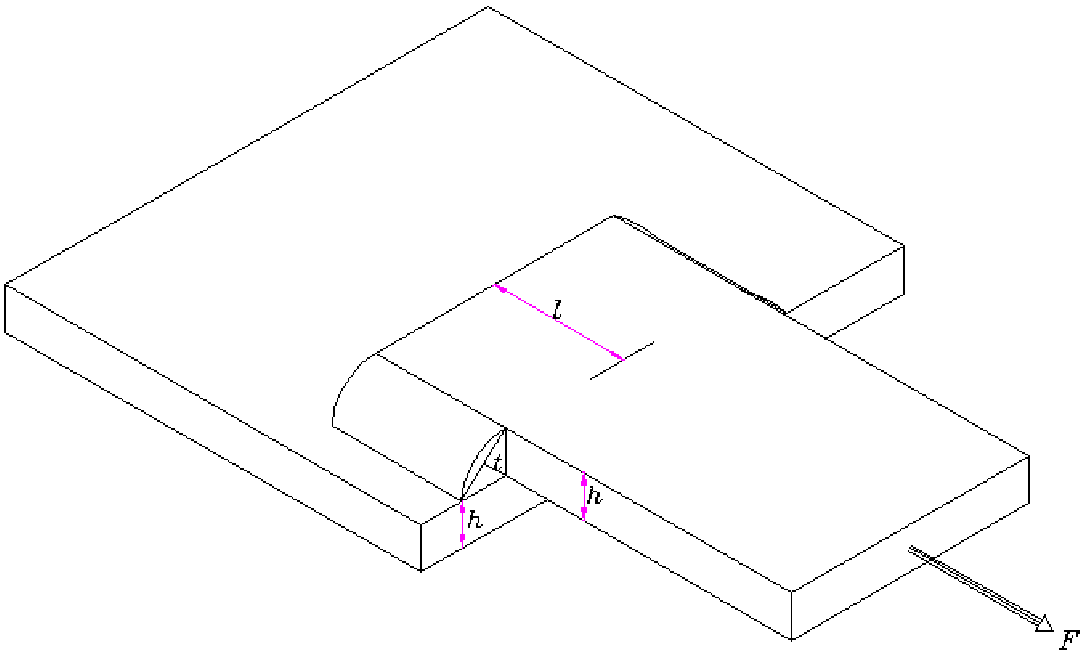
II. For double transverse fillet weld

$$\sigma_t = \frac{F}{2 \cdot A_w}$$

$$\sigma_t = \frac{F}{2 \cdot l \cdot h \cdot \sin 45}$$

B- strength of Parallel fillet weld

It is designed on the base of shear stress



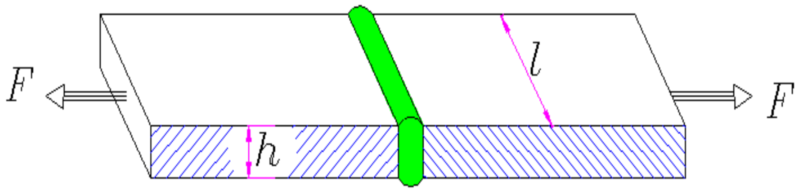
$$\tau = \frac{F}{A_w}$$

$$\tau = \frac{F}{2 \cdot l \cdot h \cdot \sin 45}$$

② strength of butt welds joints

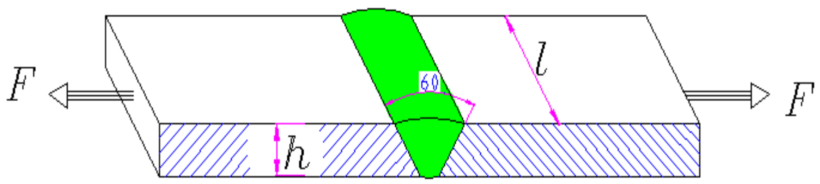
A – for Square Butt weld

$$\sigma_t = \frac{F}{A_w} = \frac{F}{h \cdot l}$$



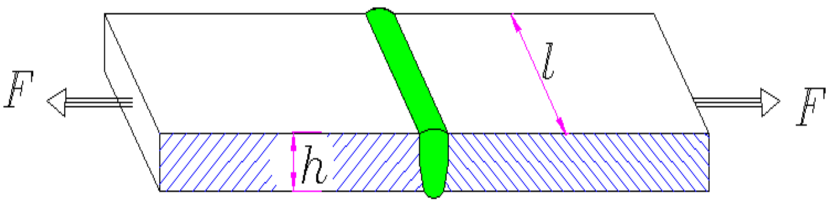
B – for Single V Butt weld

$$\sigma_t = \frac{F}{A_w} = \frac{F}{h \cdot l}$$



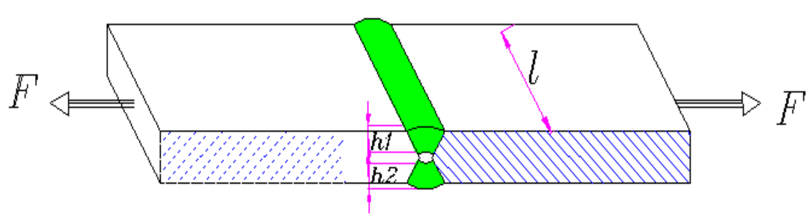
C – for U Butt weld

$$\sigma_t = \frac{F}{A_w} = \frac{F}{h \cdot l}$$



D – for Double V Butt weld

$$\sigma_t = \frac{F}{A_w} = \frac{F}{(h_1 + h_2) \cdot l}$$

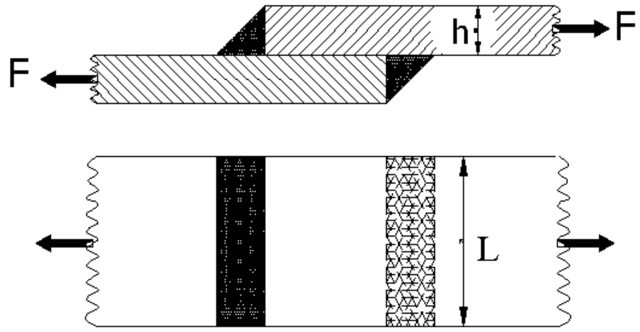


Example (1)

Two plates are jointed by double transverse fillet (lap) weld. if the allowable tension stress for weld material **(σ_t =105 N/mm²),** *plates thickness* **(6 mm)** and *the total length of weld is* **(l =100 mm)** *, determine the maximum load of weld joint.*

Solution:

$$\sigma_t = \frac{F}{A_w} = \frac{F}{2 \cdot l \cdot h \cdot \sin 45}$$



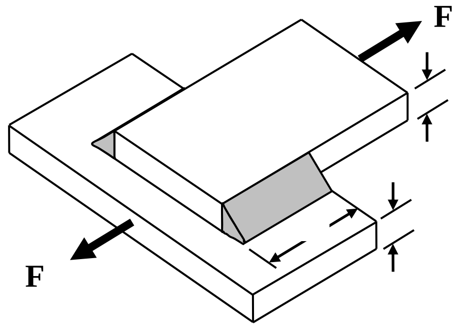
$$F = 2 \cdot l \cdot h \cdot \sin 45 \cdot \sigma_t$$

$$F = 2 \times 100 \times 6 \times \sin 45 \times 105 = 89095\, N = 89.1 KN$$

Example (2)

Determine the length of the parallel fillet lap welding required for joining two plates with thickness **(10 mm)**, *if the allowable load is* **(F=40 KN)** *and the shear stress of weld material* **(τ =80 MN/m²)**.

Solution:



$$\tau = \frac{F}{A_w}$$

$$\tau = \frac{F}{2 \cdot l \cdot h \cdot \sin 45}$$

$$l = \frac{F}{2 \cdot h \cdot \sin 45 \cdot \tau} = \frac{40 \times 10^3}{2 \times 0.01 \times \sin 45 \times 80 \times 10^6}$$

$$l = 0.0354m = 35.4mm$$

Example (3)

A spherical gas tank is made of ***(1 cm)*** *steel plate hemispheres butt weld together the tank is* ***(1500 cm)*** *in diameter. Determine the allowable internal pressure to which the tank may be subjected if the permissible stress be limited to* ***(84 MN/m²)***

Solution:

$$A_w = l \cdot h = \pi \cdot d \cdot h = \pi \times \frac{1500}{100} \times \frac{1}{100} = 0.47m^2$$

Bursting load resisted by the weld

$$\sigma_t = \frac{F}{A_w}$$

$$F = \sigma_t \cdot A_w = 84 \times 10^6 \times 0.47 = 39480000 .N = 39.48 \times 10^6 N$$

Let the gas pressure is P

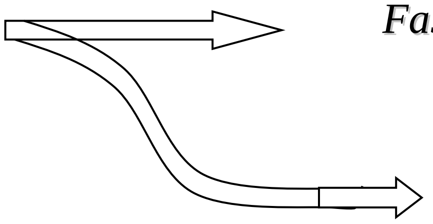
$$P = \frac{F_{bur}}{A} \Longrightarrow F_{bur} = P \cdot A = P \times \frac{\pi}{4} \times (15)^2$$

$$\frac{\pi}{4} \times (15)^2 \times P = 39.48 \times 10^6$$

$$P = \frac{39.48 \times 10^6}{\frac{\pi}{4} \times (15)^2} = 223411 \frac{N}{m^2}$$

Screws

Types of screws



Fastening screw (bolts)

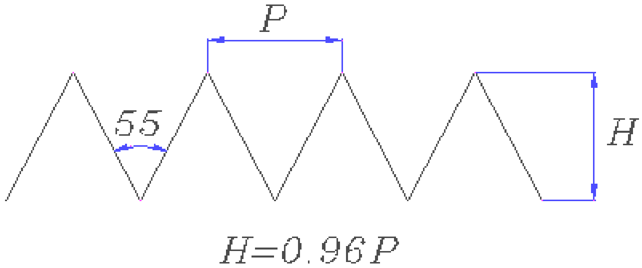
Power screw

Fastening screw (joints)

Types of screw teeth

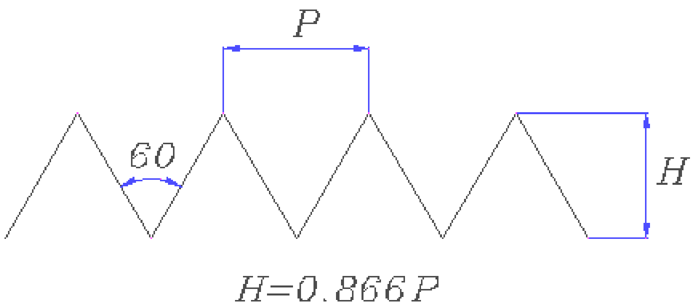
1- British standard

Angle of tooth =55°



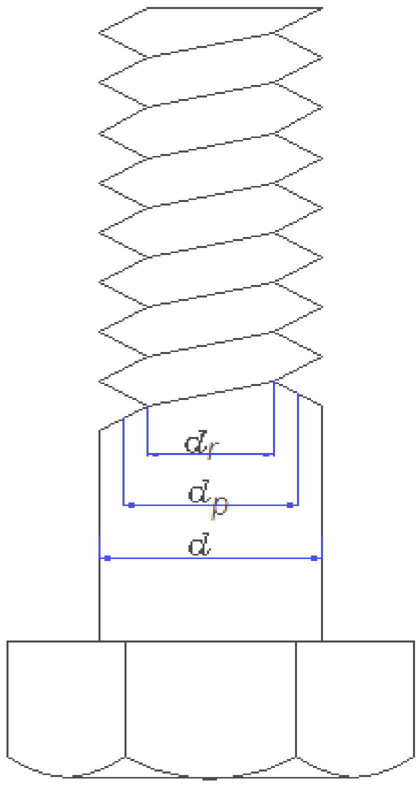
2-American national standard

Angle of tooth =60°



Definitions

d_i : inside diameter :root diameter (d_r)
d_p: pitch diameter.
d : outside diameter : nominal diameter.



Design of bolt:

The bolts are subjected to these stresses

1- tensile stress

2- shear stress

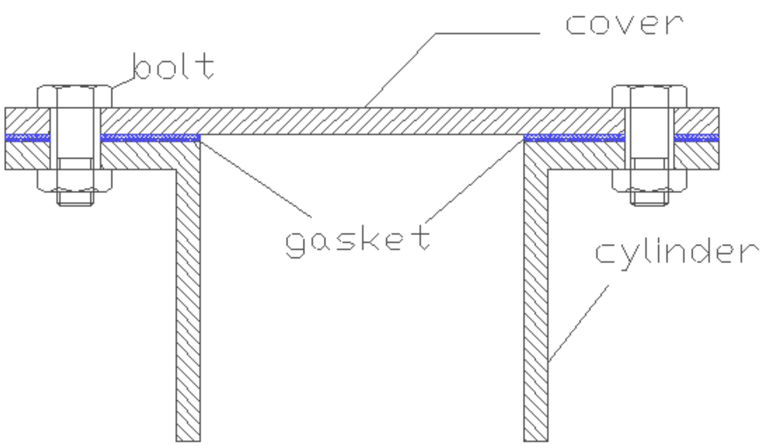
3- torsion stress

but in practice we cannot determine all these stresses with high accuracy so we calculate the screw only one the base of direct tensile stress and take the appropriate factor of safety

$$S.F = (2 \rightarrow 2.5)$$

Design bolts for tension :

The total force act on the screw can be calculated by



$$F_t = F_i + K \cdot F_e$$

$$F_i = 2840 \times d$$

$$F_e = P \cdot A$$

F_i: initial tension on bolt in ***(N)***

d: nominal diameter of bolt ***(mm)***

K: stiffness constant (calculated from table 1

F_e: external force ***(N)***

P: pressure inside the cylinder ***(N/m²)***

A: cross section area of cylinder **(m^2)**

the value of "K" determined from table(1) and depend on the joint type and gasket type.

Table (1)

	Type of joint and gasket	***Value of "K"***
1	*Soft gasket with stud*	1
2	*Soft gasket with through bolt*	0.75
3	*Asbestos gasket with through bolt*	0.6
4	*Copper gasket with through bolt*	0.5
5	*Hard Copper gasket with through bolt*	0.25
6	*With out gasket by using stud*	0.1
7	*With out gasket by using through bolt*	0

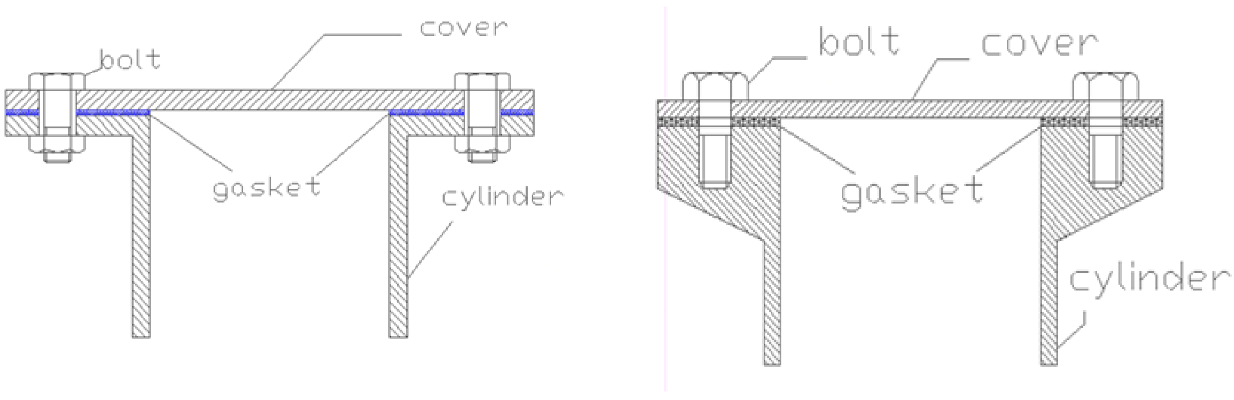


Table (2) size of bolts

size of bolt d(mm)	*pitch p(mm)*	*Root section area (mm^2)*
M1.6	0.35	1.27
M2	0.4	2.07
M2.5	0.45	3.39
M3	0.5	5.03
M4	0.8	14.2
M6	1	20.1
M8	1.25	36.6
M10	1.5	58
M12	1.75	84.3
M16	2	157
M18	2.5	192
M20	2.5	245
M22	2.5	303
M24	3	353
M27	3	459
M30	3.5	561
M33	3.5	694
M36	4	817
M39	4	976
M42	4.5	1120
M48	5	1470

Tensile stress act on the screw

$$\sigma_t = \frac{F_t}{A_r}$$

Design bolts for shear:

Shear stress act on the screw

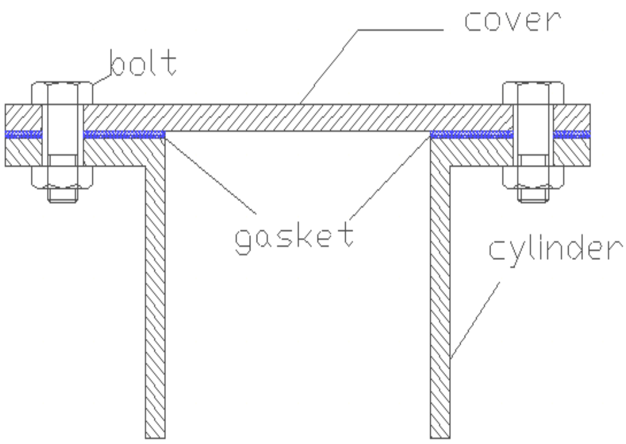
$$\tau = \frac{F}{A_r}$$

F: shear force **(N)**

A_r: root section area for bolt determine from table (2) according to the size of bolt.

Example (1)

Cylinder cover of internal combustion engine is jointed by 10 through bolts. if the diameter of the cylinder ***(30 cm)****, gases pressure* ***(80 KN/m²)*** *tension stress for the bolt material is* ***(100 MN/m²). Determine the size of bolts.***



Solution:

$$F_e = P \cdot A = 80000 \times \frac{\pi}{4}(0.3)^2 = 5655.N$$

$$External\ force\ for\ one\ bolt = \frac{5655}{10} = 565.5N$$

From table (1) K=0.75

$$F_i = 2840 \times d$$

$$F_t = F_i + K \cdot F_e = 2840 \times d + 0.75 \times 565.5$$

$$F_t = 2840 \times d + 424$$

$$\sigma_t = \frac{F_t}{A_r} \quad \Longrightarrow \quad 100 = \frac{2840 \times d + 424}{\frac{\pi}{4} \times (d_r)^2}$$

$78.5(d_r)^2 - 2840 d_r - 424 = 0$

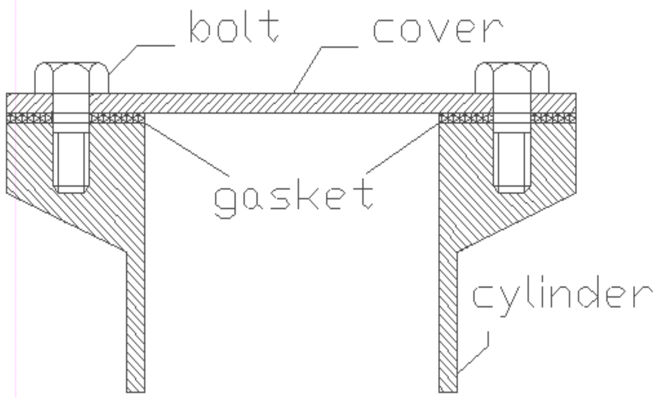
$(d_r)^2 - 36.2 d_r - 5.4 = 0$

$d_r = 36.3mm$ $A_r = \frac{\pi}{4} \times (d_r)^2 = 1037.7mm^2$

From table ② the size of bolt is M42

Example (2)

The cylinder head of steam engine is held in position by ***(12)*** *studs. the cylinder bore is* ***(500 mm)*** *and the max. pressure is* ***(12 kg/cm²).*** *if the tensile stress for stud is* ***(1000 kg/cm²)*** *and the stiffness coefficient* ***(K=0.25).*** *Determine the size of the studs.*



Assume the initial force. $F_i = 2840 \times d$

Solution:

D = 500 mm

*P =12*10/100=1.2 N/mm²*

$$\sigma_t = \frac{1000 \times 10}{100} = 100 \frac{N}{mm^2}$$

$$F_e = P \cdot A = 1.2 \times \frac{\pi}{4} (500)^2 = 235619.4N$$

$$F_e \quad per \quad stud = \frac{235619.4}{12} = 19635N$$

$$F_t = F_i + K \cdot F_e = 2840 \times d + 0.25 \times (19635)$$

$$F_t = 2840 \times d + 4908.7$$

$$\sigma_t = \frac{F_t}{A_r} \Longrightarrow 100 = \frac{2840 \times d + 4908.7}{\frac{\pi}{4} \times (d_r)^2}$$

$$78.5(d_r)^2 - 2840 \times d_r - 4908.7 = 0$$
$$(d_r)^2 - 36.2 d_r - 62.5 = 0$$

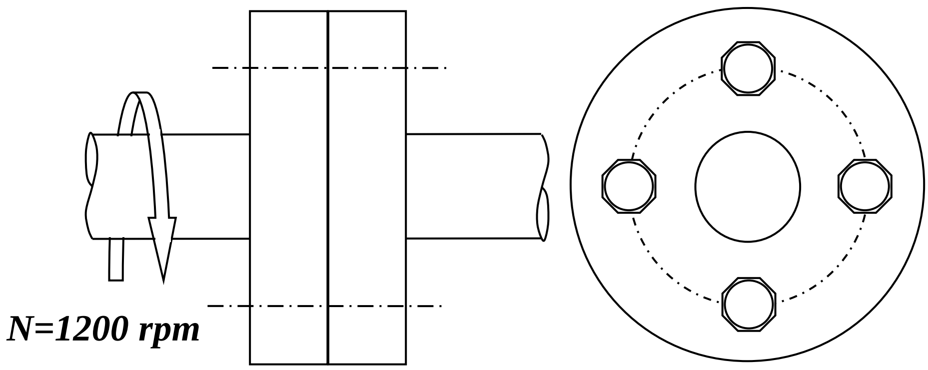
$$d_r = 37.9mm$$
$$d_r = -1.7mm$$

$$\therefore d_r = 37.9mm \qquad A_r = \frac{\pi}{4} \times (d_r)^2 = 1128.2mm^2$$

From table (2) the size of bolt is M_{42}

Example (3)

*Rigid coupling translate power **(15 KW)** and turn with speed **(1200 rpm)** as shown in fig. if the driver and driven parts are fastened by four bolts arranged on the circumference of circle with diameter of* **(12 cm)** and *the shear stress for every bolt **(τ = 15 MN/m²** **)**, determine the size of the bolts.*



Solution:

Angular speed $w = \frac{2 \cdot \pi \cdot N}{60} = \frac{2 \times \pi \times 1200}{60}$

$$w = 125.7 \frac{rad}{\sec}$$

$$Power = P = T \cdot w$$

$$15000 = T \times 125.7$$
$$T = 119.4.N \cdot m$$

$$T = F \cdot R \cdot n$$

$$119.4 = F \times 0.06 \times 4 \quad \Longrightarrow \quad F = 497.4N$$

$$\tau = \frac{F}{A_r}$$

$$15 \times 10^6 = \frac{497.4}{A_r}$$
$$A_r = 3.315 \times 10^{-5} m^2$$
$$A_r = 33.15 mm^2$$

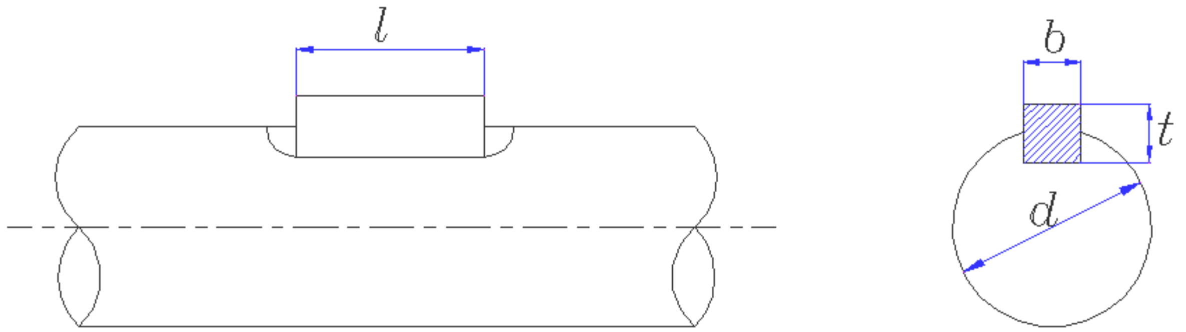
From table (2) we find d = 8 mm so the size of bolt is M_8

Keys:

The function of a key is to prevent relative rotation of a shaft and the hub [such as gear, pulley] it is non permanent joint.

Types of key:

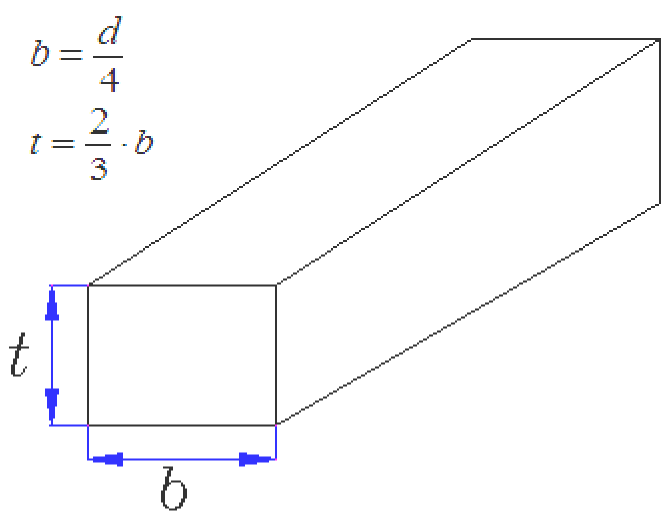
(1) sunk key: *in this type half of the key is sunken in the shaft and the other in the hub [gear or pulley]*



There types are:

A - Rectangular sunk key:

Sunk key with rectangular cross section may be of uniform cross section or may be tapered by 1:100 for easy and stable connection



Where:

b: width

d: diameter of shaft

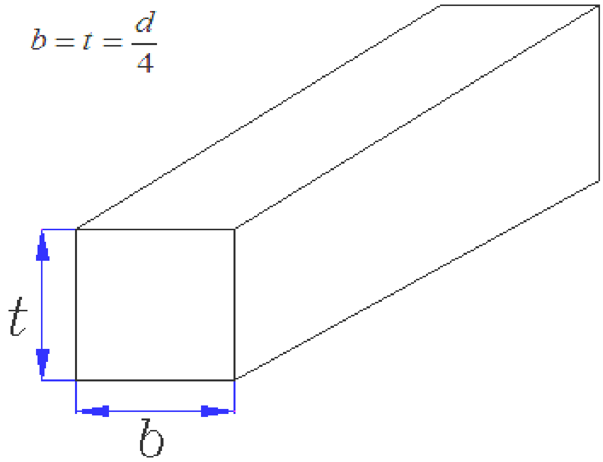
t: thickness

l: length of key

B - Square sunk key:

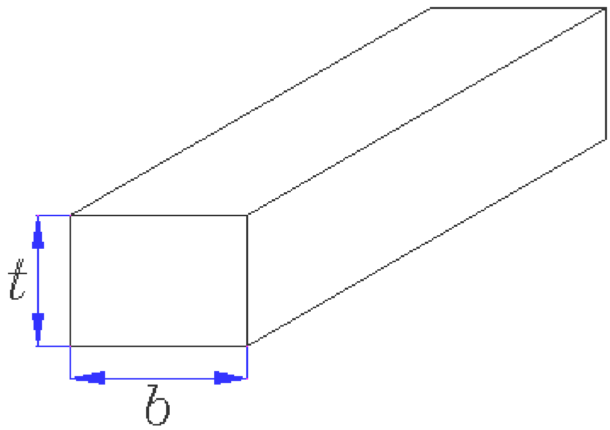
This type has the same width and depth

$$b = t = \frac{d}{4}$$



C - Parallel sunk key:

The cross section of this type may be rectangular or square and is used for gear and pulley which are turn and slide over the shaft in the same time, this type is untapered.

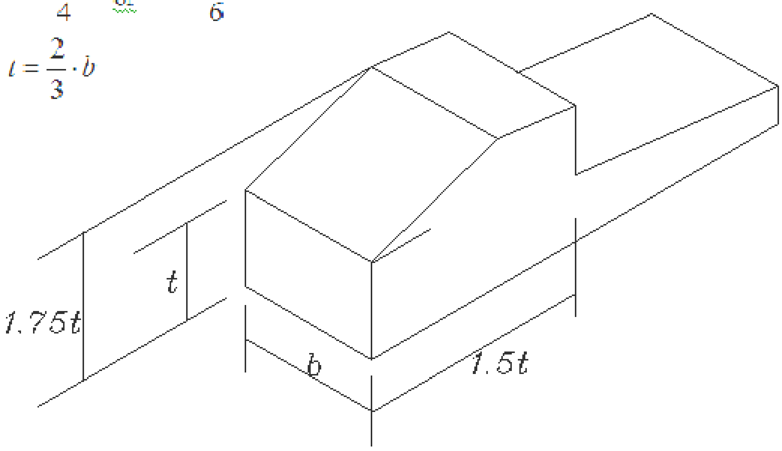


D - Gib head sunk key:

It is a tapered key with rectangular cross section area and has gib head for easy installation.

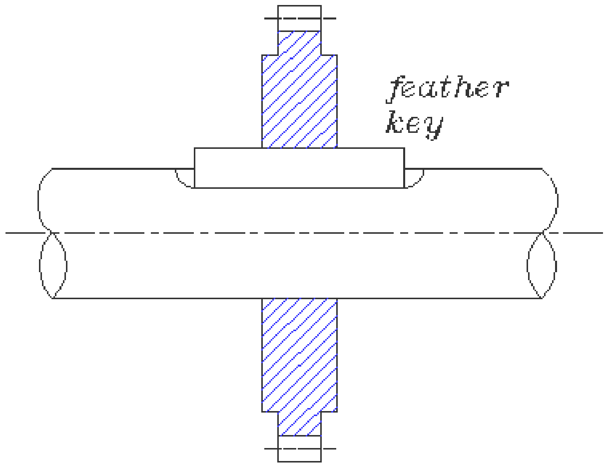
$$b = \frac{d}{4} \quad \text{or} \quad b = \frac{d}{6}$$

$$t = \frac{2}{3} \cdot b$$



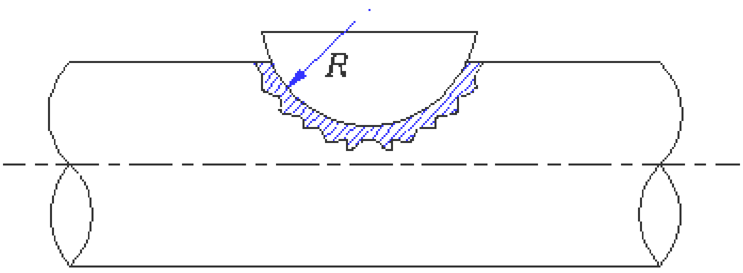
E - Feathers key:

This is special type used to transmit turning moment and allow for axial motion it joints on the shaft or pulley by tapered pin or it has two heads



F - Wood ruff sunk key:

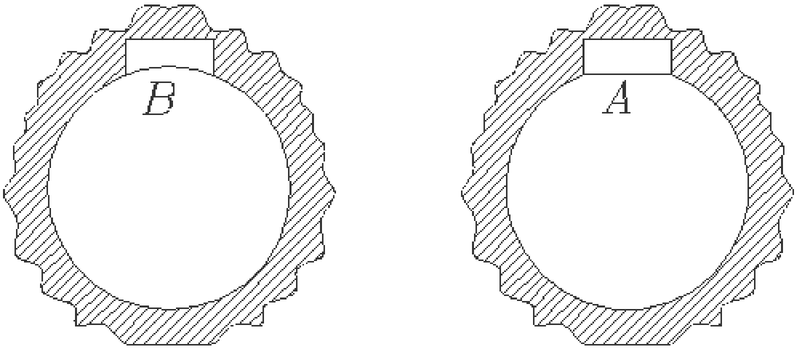
It has the form of semi-circle and requires a special side milling cutter to form the key seat. disadvantage of this key is weaking the shaft and it is used in automobile joints.



(2) Saddle keys:

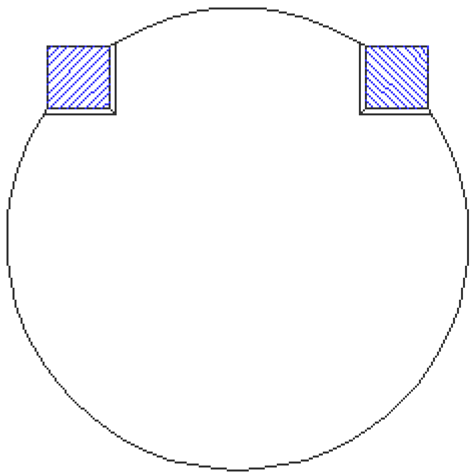
A - Flat saddle key: used for light load.

B - Hollow saddle key



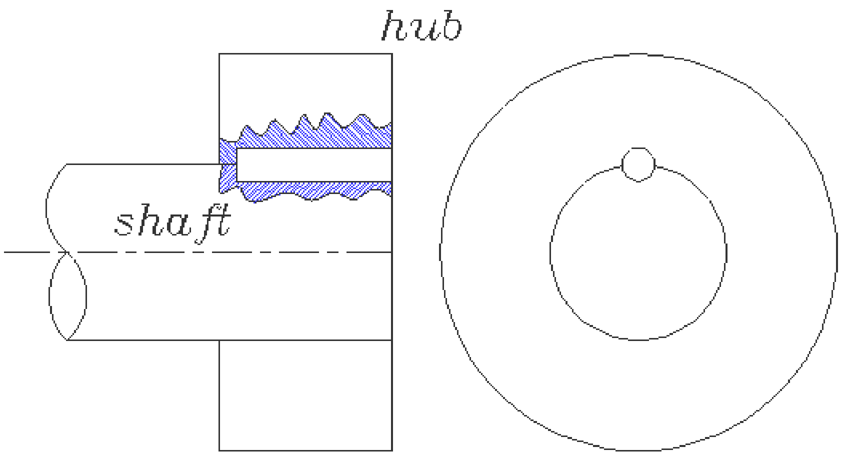
③ tangent keys:

Used for heavy duty and consists of two key which are fixed on the shaft with right angle every key bear the turning moment in one direction.



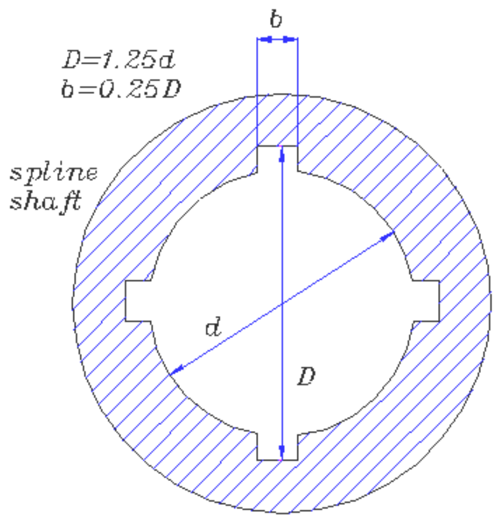
④ Round key :

This type has a circular cross section area and is fixed in a hole which is drilled between shaft and hub. this type is used for light duty.



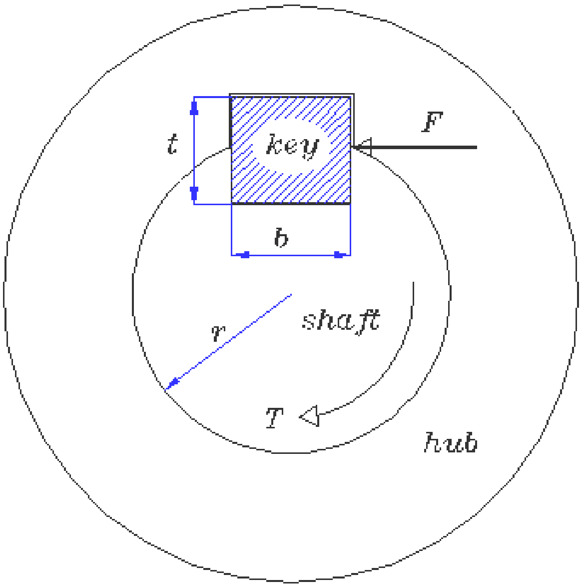
⑤ Splines :

Used for heavy duty. It is composed of a splined shaft formed by milling and mating hub with internal splines.



Design of sunk key:

The key is subjected to shear stress and bearing stress due to the tangential force.



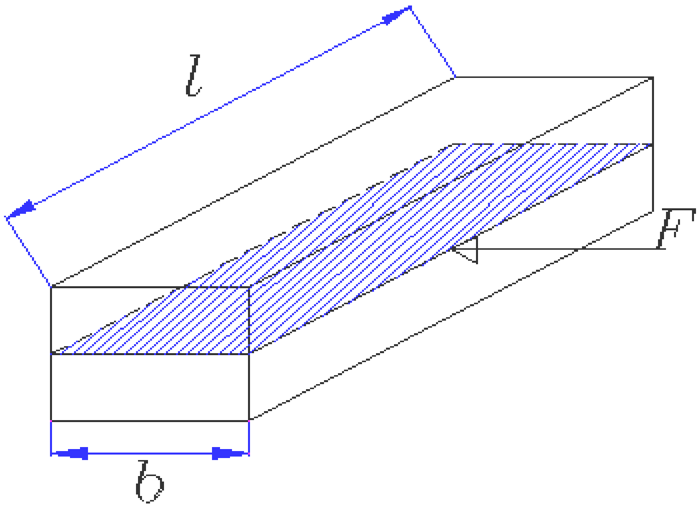
1 - According to shear stress:

$$\tau = \frac{F}{A} = \frac{F}{b \cdot l}$$

$$\because T = F \cdot \frac{d}{2}$$

$$\therefore F = \frac{2 \cdot T}{d}$$

$$\tau = \frac{2 \cdot T}{d \cdot b \cdot l}$$

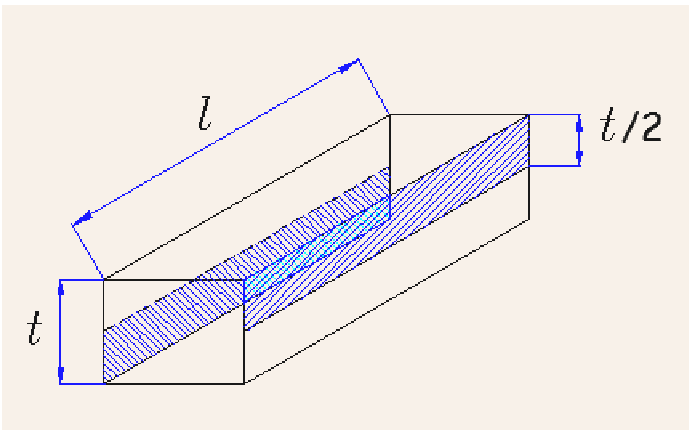


2 - According to bearing or (crushing) stress:

$$\sigma_{bearing} = \frac{F}{A_{proj}} = \frac{F}{\frac{t}{2} \cdot l}$$

$$\because T = F \cdot \frac{d}{2} \qquad \therefore F = \frac{2 \cdot T}{d}$$

$$\sigma_{bearing} = \frac{2 \cdot T}{\frac{t}{2} \cdot l \cdot d}$$



Example (1)

*Determine the dimensions of the rectangular sunk key. If the diameter of the shaft **(d =40 mm),** transmitted force **(F = 20 KN)**, thickness of key **(t= 10 mm)**, permissible bearing stress **($\sigma_{bearing}$ = 250 MN/m²)*** and the shear stress ***(τ =104 MN/m²)***.

Solution :

$$\sigma_{bearing} = 250 \times 10^6 \frac{N}{m^2}$$

$$\tau = 104 \times 10^6 \frac{N}{m^2}$$

$$F = 20 \times 10^3 N$$

$$t = \frac{10}{1000} = 0.01m$$

$$\sigma_{bearing} = \frac{F}{A_{proj}} = \frac{F}{\frac{t}{2} \cdot l}$$

$$250 \times 10^6 = \frac{20 \times 10^3}{\frac{0.01}{2} \times l} \Longrightarrow l = 0.016m$$

$$\therefore l = 16mm$$

$$\tau = \frac{F}{A} = \frac{F}{b \cdot l}$$

$$104 \times 10^6 = \frac{20 \times 10^3}{b \times 0.016} \Longrightarrow b = 0.012m$$

$$\therefore b = 12mm$$

Example (2)

*A rectangular sunk key with dimension **L*b*t=75*14*10mm** is required to transmit a torque **(T = 1200 N.m)** from a solid shaft of diameter **(d = 50 mm)** determine weather the length is sufficient or not , if the permissible shear stress and crushing stress are (τ = 56 MN/m^2), (σ_{crush} = 168 MN/m^2).*

Solution :

$$l = \frac{75}{1000} = 0.075m \quad , \quad b = \frac{14}{1000} = 0.014m \quad , \quad t = \frac{10}{1000} = 0.01m$$

$$T = 1200\,N \cdot m \quad , \quad d = \frac{50}{1000} = 0.05m \quad , \quad \tau_{per} = 56 \times 10^6 \frac{N}{m^2}$$

$$\sigma_{crush)per} = 168 \times 10^6 \frac{N}{m^2}$$

$$\tau_{cal} = \frac{2 \cdot T}{d \cdot b \cdot l} = \frac{2 \times 1200}{0.05 \times 0.014 \times 0.075} = 45714286 \frac{N}{m^2}$$

$$\tau_{cal} = 45.7 \frac{MN}{m^2} \quad < \quad \tau_{per} = 56 \frac{MN}{m^2} \qquad \text{O.K}$$

$$\sigma_{crush)cal} = \frac{2 \cdot T}{\frac{t}{2} \cdot l \cdot d} = \frac{2 \times 1200}{\frac{0.01}{2} \times 0.075 \times 0.05} = 128000000 \frac{N}{m^2}$$

$$\sigma_{crush)cal} = 128 \frac{MN}{m^2} \quad < \quad \sigma_{crush)per} = 168 \frac{MN}{m^2} \qquad \text{O.K}$$

the length of key is sufficient

Example (3)

A belt pulley transmitting power is secured to a steel shaft ***(50 mm)*** *diameter. The key provided has a width of* ***(18mm)*** *and a thickness of* ***(16mm)*** *and is* ***(125mm)*** *long. the material of the key allows a stress of* ***(400kg/cm²)*** *in shear and* ***(950kg/cm²)*** *in bearing. what h.p can be transmitted by the pulley when running at* ***(200rpm).***

Solution:

$$d = \frac{50}{1000} = 0.05m \quad , \quad b = \frac{18}{1000} = 0.018m \quad , \quad t = \frac{16}{1000} = 0.016m$$

$$l = \frac{125}{1000} = 0.125m \quad , \quad \tau = 400\,\frac{kg}{cm^2} = \frac{400\times 10}{10^{-4}} = 40000000\,\frac{N}{m^2}$$

$$\sigma_{bearing} = 950\,\frac{kg}{cm^2} = \frac{950\times 10}{10^{-4}} = 95000000\,\frac{N}{m^2} \quad , \quad N = 200.rpm$$

$$\tau = \frac{F}{A} = \frac{F}{b\cdot l} \Longrightarrow 40\times 10^6 = \frac{F}{0.018\times 0.125}$$

$$F = 90000\,N$$

$$\sigma_{bearing} = \frac{F}{A_{proj}} = \frac{F}{\frac{t}{2}\cdot l} \Longrightarrow 95\times 10^6 = \frac{F}{\frac{0.016}{2}\times 0.125}$$

$$F = 95000\,N$$

$$\therefore F = 90000\,N$$

$$T = F\cdot\frac{d}{2} = 90000\times\frac{0.05}{2} = 2250\,N\cdot m$$

$$power = T\cdot w = 2250\times\left(\frac{2\times\pi\times 200}{60}\right) = 47124.watt$$

$$power = 47124 \div 746 = 63.2.h.p$$

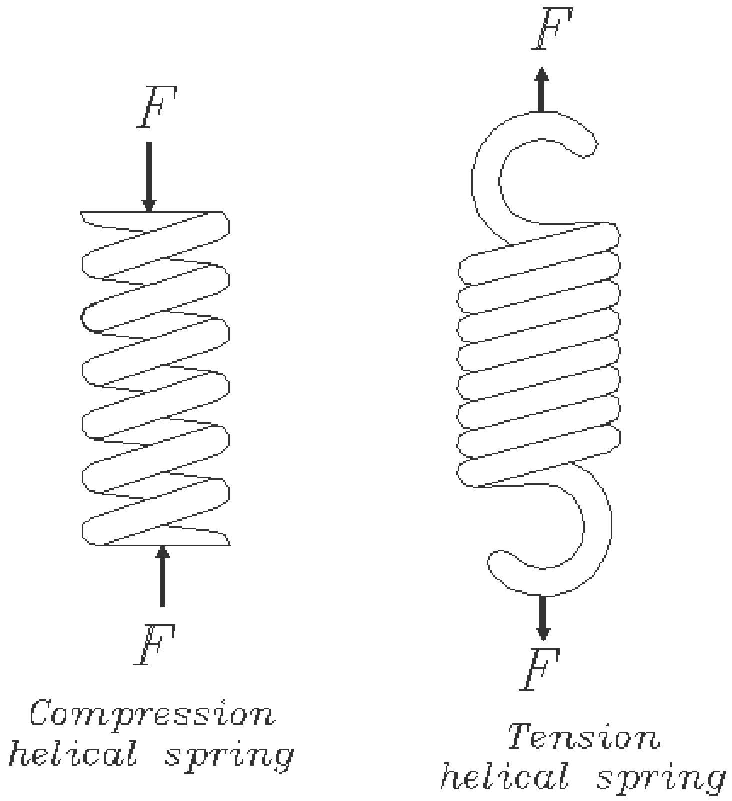
Springs:

It defined as an elastic body whose function is to deflect under load and when the load is released it return to its original shape.

Types of springs:

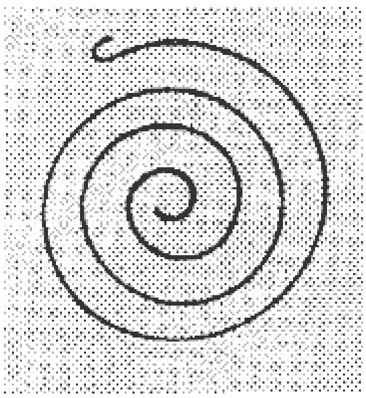
(I) - Helical spring:

It is made of wire coiled in to helical form and is subjected to tensile force or compressive force



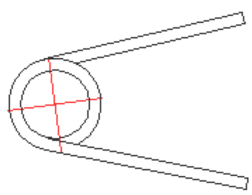
(II)-Helical torsion spring:

Also, it is made of wire coiled into a helical form and subjected to torsion moment.



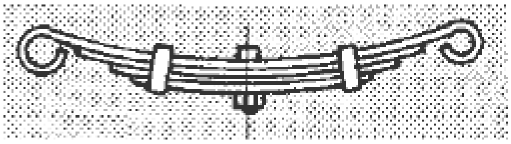
(III) -Spiral springs:

It consists of flat strip wound in the form of spiral and is subjected to torsion moment.



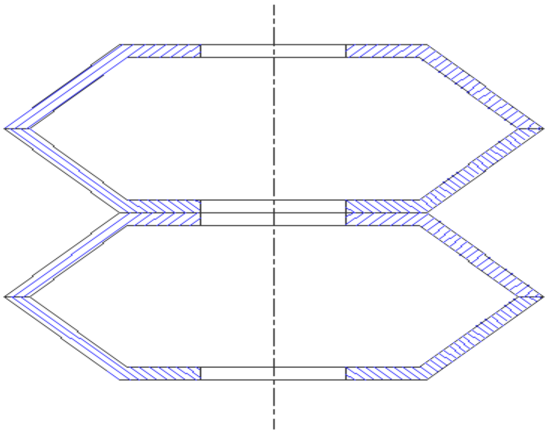
(IV) -Leaf spring:

It composed of flat bars of varying lengths clamped to gather and are subjected to load.

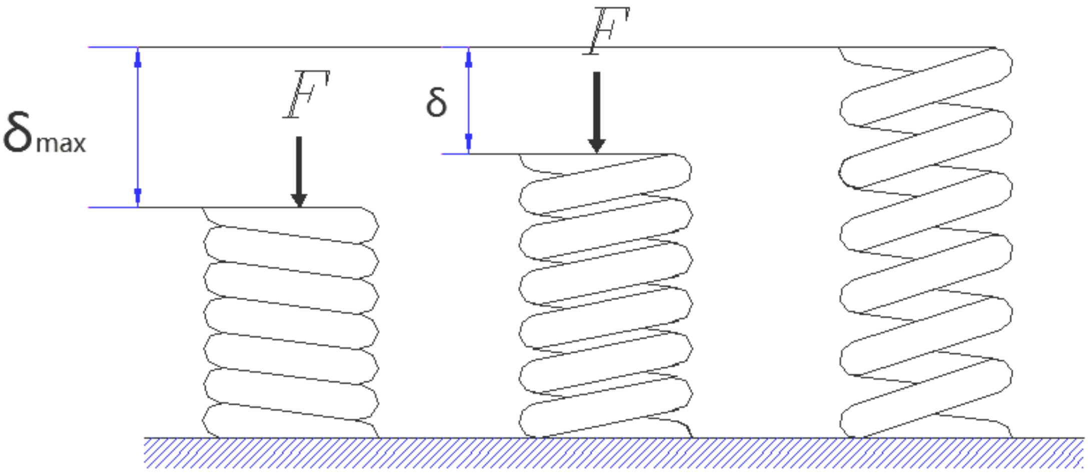


(V) -Belleville spring:

They are composed of coned discs and are subjected to compressive force.



Definition of helical spring:



1-Solid length: L_s

It is the length of spring when it is compressed to form hollow cylinder

$$L_s = n \cdot d$$

Where n: number of active coils
d: diameter of spring wire

2-Free length: L_f

Length of spring when it is not subjected to external force.

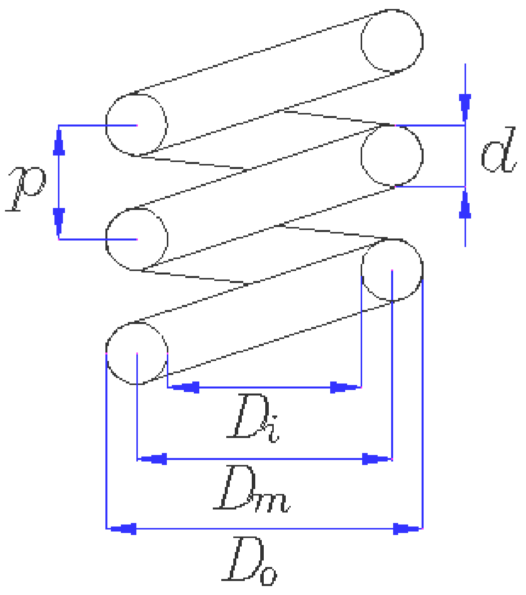
$$L_f = L_s + \delta_{max} + \varepsilon$$

$$L_f = n \cdot d + \delta_{max} + (n-1) \times 0.1$$

Where δ : *deflection of spring*
δ_{max} : *max. deflection of spring*
ε : *spring end coils*

3- Spring index (C):

It is the ratio between the mean diameter of coil spring and the diameter of spring wire



$$C = \frac{D_m}{d}$$

$$D_m = D_i + d$$
$$D_m = D_o - d$$

4- Spring rate (stiffness) (K):

It is defined as a force which is required to extend or expand the spring to one unit length.

$$K = \frac{F}{\delta} \; \frac{N}{m}$$

Where F : *external force (N)*
δ : *Deflection of spring (m)*

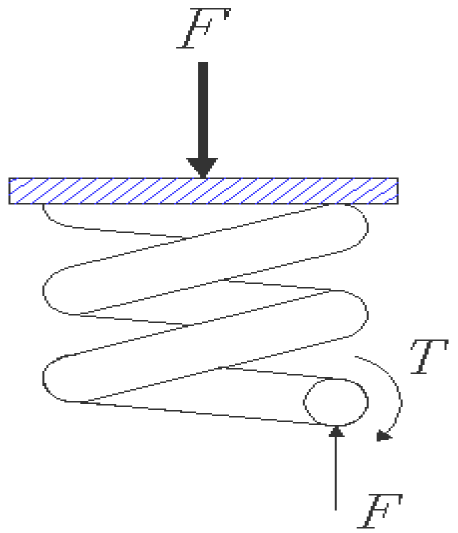
5-pitch of coil spring:

It is the distance between any two coils when the spring is free

$$p = \frac{L_f}{(n-1)}$$

Design of helical spring:

Helical spring is designed on the base of shear stress due to torsion and direct shear (neglected the curvature effect)



I –Direct shear stress

$$\tau_d = \frac{F}{\frac{\pi}{4} \cdot d^2} = \frac{4 \cdot F}{\pi \cdot d^2}$$

II-Torsion shear stress

$$\tau_t = \frac{T \cdot r}{J} = \frac{F \cdot \frac{D_m}{2} \cdot \frac{d}{2}}{\frac{\pi}{32} \cdot d^4} = \frac{8 \cdot F \cdot D_m}{\pi \cdot d^3}$$

$$\tau = \tau_d + \tau_t = \frac{4 \cdot F}{\pi \cdot d^2} + \frac{8 \cdot F \cdot D_m}{\pi \cdot d^3}$$

$$\tau = \frac{8 \cdot F \cdot D_m}{\pi \cdot d^3}\left(\frac{d}{2 \cdot D_m} + 1\right)$$

$$\because C = \frac{D_m}{d}$$

$$\tau = \frac{8 \cdot F \cdot D_m}{\pi \cdot d^3}\left(\frac{1}{2 \cdot C} + 1\right)$$

By taking curvature effect

$$\tau = \frac{8 \cdot k \cdot F \cdot D_m}{\pi \cdot d^3} \qquad or \qquad \boxed{\tau = \frac{8 \cdot k \cdot F \cdot C}{\pi \cdot d^2}}$$

$$\boxed{k = \frac{4 \cdot C - 1}{4 \cdot C - 4} + \frac{0.615}{C}}$$

Calculation of spring deflection

1-Angular deflection: θ

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G \cdot \theta}{L} \qquad \theta = \frac{T \cdot L}{J \cdot G}$$

$$\because T = F \cdot \frac{D_m}{2}$$

$$J = \frac{\pi}{32} \cdot d^4$$

$$L = \pi \cdot D_m \cdot n$$

$$\theta = \frac{F \cdot \frac{D_m}{2} \cdot \pi \cdot D_m \cdot n}{\frac{\pi}{32} \cdot d^4 \cdot G} = \frac{16 \cdot F \cdot D_m^2 \cdot n}{d^4 \cdot G}$$

2-Axial deflection: δ

$$\delta = \theta \cdot \frac{D_m}{2} = \frac{16 \cdot F \cdot D_m^2 \cdot n}{d^4 \cdot G} \times \frac{D_m}{2}$$

$$\delta = \frac{8 \cdot F \cdot D_m^3 \cdot n}{d^4 \cdot G}$$

$$\delta = \frac{8 \cdot F \cdot C^3 \cdot n}{d \quad \cdot G}$$

Example (1)

Helical spring with outside diameter ***(D_0=7.5cm)*** *and manufactured from wire with diameter* ***(d = 6mm)*** *if the shear stress of wire material* ***(τ = 350 MN/m²)*** *and the modulus of rigidity* ***(G = 84 GN/m²).****determine the external (axial) load and spring (axial) deflection per coil .*

Solution:

$$D_o = 7.5cm = \frac{7.5}{100} = 0.075m \qquad , \qquad d = 6mm = \frac{6}{1000} = 0.006m$$

$$\tau = 350 \times 10^6 \frac{N}{m^2} \qquad , \qquad G = 84 \times 10^9 \frac{N}{m^2}$$

$$D_m = D_o - d = 0.075 - 0.006 = 0.069m$$

$$C = \frac{D_m}{d} = \frac{0.069}{0.006} = 11.5$$

$$k = \frac{4 \cdot C - 1}{4 \cdot C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.125$$

$$\tau = \frac{8 \cdot k \cdot F \cdot C}{\pi \cdot d^2} \Longrightarrow F = \frac{\tau \cdot \pi \cdot d^2}{8 \cdot k \cdot C}$$

$$F = \frac{350 \times 10^6 \times \pi \times (0.006)^2}{8 \times 1.125 \times 11.5} = 382.5N$$

$$\delta = \frac{8 \cdot F \cdot C^3 \cdot n}{d \quad \cdot G}$$

$$\frac{\delta}{n} = \frac{8 \cdot F \cdot C^3}{d \cdot G} = \frac{8 \times 382.5 \times (11.5)^3}{0.006 \times 84 \times 10^9}$$

$$\frac{\delta}{n} = 0.00923m$$

Example (2)

*Helical spring with mean diameter **(D_m=25mm)** and manufactured from wire with diameter **(d=3mm).**if the shear stress of spring material **(τ =441 MN/m^2)**, axial deflection **(δ=25mm)** and the modulus of rigidity **(G =86.2 GN/m^2)**. Determine the external force and the number of active coils.*

Solution:

$$D_m = 0.025m \quad , \quad d = 0.003m \quad , \quad \tau = 441 \times 10^6 \frac{N}{m^2}$$

$$\delta = 0.025m \quad , \quad G = 86.2 \times 10^9 \frac{N}{m^2}$$

$$C = \frac{D_m}{d} = \frac{0.025}{0.003} = 8.3$$

$$k = \frac{4 \cdot C - 1}{4 \cdot C - 4} + \frac{0.615}{C} = \frac{4 \times 8.3 - 1}{4 \times 8.3 - 4} + \frac{0.615}{8.3} = 1.177$$

$$\tau = \frac{8 \cdot k \cdot F \cdot C}{\pi \cdot d^2} \Longrightarrow F = \frac{\tau \cdot \pi \cdot d^2}{8 \cdot k \cdot C}$$

$$F = \frac{441 \times 10^6 \times \pi \times (0.003)^2}{8 \times 1.177 \times 8.3} = 159.5N$$

$$\delta = \frac{8 \cdot F \cdot C^3 \cdot n}{d \cdot G} \Longrightarrow n = \frac{\delta \cdot d \cdot G}{8 \cdot F \cdot C^3}$$

$$n = \frac{0.025 \times 0.003 \times 86.2 \times 10^9}{8 \times 159.5 \times (8.3)^3} = 8.9 \approx 9$$

Example (3)

Design helical compression spring to carry load (F=1000N) and having axial deflection (δ =25mm) spring index (C = 5), shear stress ***(τ =420 MN/m²)****, and the modulus of rigidity* ***(G =86.2 GN/m²)****.*

Solution:

$F = 1000N$, $\delta = 0.025m$, $C = 5$, $\tau = 420 \times 10^6 \frac{N}{m^2}$

$G = 84 \times 10^9 \frac{N}{m^2}$

1- Diameter of wire (d)

$$k = \frac{4 \cdot C - 1}{4 \cdot C - 4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

$$\tau = \frac{8 \cdot k \cdot F \cdot C}{\pi \cdot d^2} \Longrightarrow d = \sqrt{\frac{8 \cdot k \cdot F \cdot C}{\pi \cdot \tau}}$$

$$d = \sqrt{\frac{8 \times 1.31 \times 1000 \times 5}{\pi \times 420 \times 10^6}} = 0.0063m = 6.3mm$$

2- Outside and inside diameter D_o ,D_i

$$C = \frac{D_m}{d} \Longrightarrow D_m = C \cdot d = 5 \times 6.3 = 31.5mm$$

$$D_o = D_m + d = 37.8mm$$

$$D_i = D_m - d = 25.2mm$$

3- Number of active coils n

$$\delta = \frac{8 \cdot F \cdot C^3 \cdot n}{d \quad \cdot G} \Longrightarrow n = \frac{\delta \cdot d \cdot G}{8 \cdot F \cdot C^3}$$

$$n = \frac{0.025 \times 0.0063 \times 84 \times 10^9}{8 \times 1000 \times (5)^3} = 13.23 \qquad \therefore n = 14$$

4-solid length L_s

$$L_s = n \cdot d = 14 \times 6.3 = 88.2mm$$

5-free length L_f

$$L_f = n \cdot d + \delta_{\max} + (n-1) \times 0.1$$

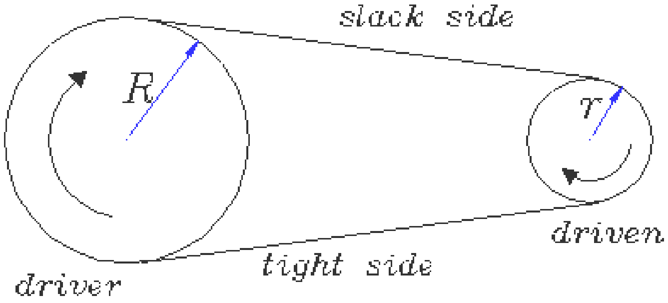
$$L_f = 14 \times 6.3 + 25 + (14-1) \times 0.1 = 114.5mm$$

6-pitch p

$$p = \frac{L_f}{(n-1)} = \frac{114.5}{13} = 8.81mm$$

Belts:

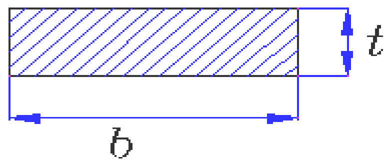
The belts are used to transmit the power from one shaft to another when the distance between the shaft axes is large and the angular velocity ratio of the driving and driven member is not constant or allow slip.



Types of Belts:

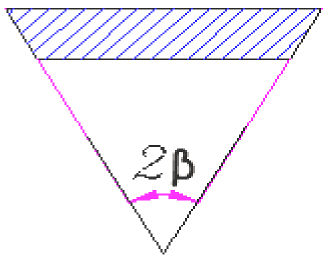
1) Flat belt:

It is used to transmit high power with low speed.



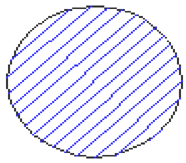
2) V- belt:

It is used to transmit power more than flat belt.



3) Circular belt (rope):

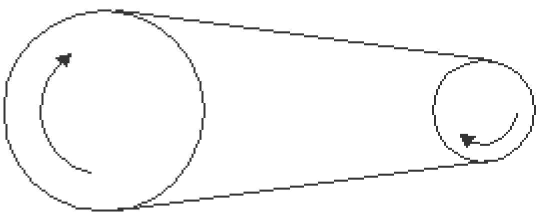
It is used to transmit high power with high speed.



Types of belt drives:

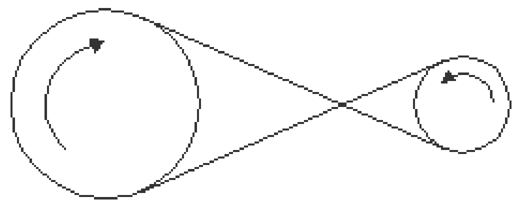
(I) Open belt drive

This type is used to transmit power between two parallel axis and turning in same direction.



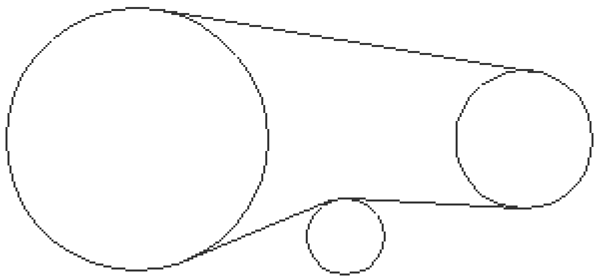
(II) *Cross belt drive*

It is used for transmitting power between two parallel axes, turn in opposite direction.



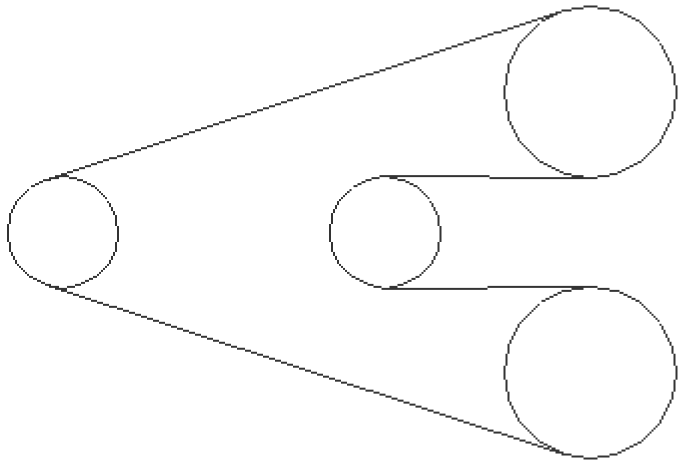
(III) *belt drive with idler pulley*

Idler pulley is used to increase are of contact and belt tension.



(IV) *Belt drive with many pulleys*

It is used to transmit power from one shaft to many shafts with one belt.



Design of belts:

1- Velocity ratio:

(A) By neglecting slip

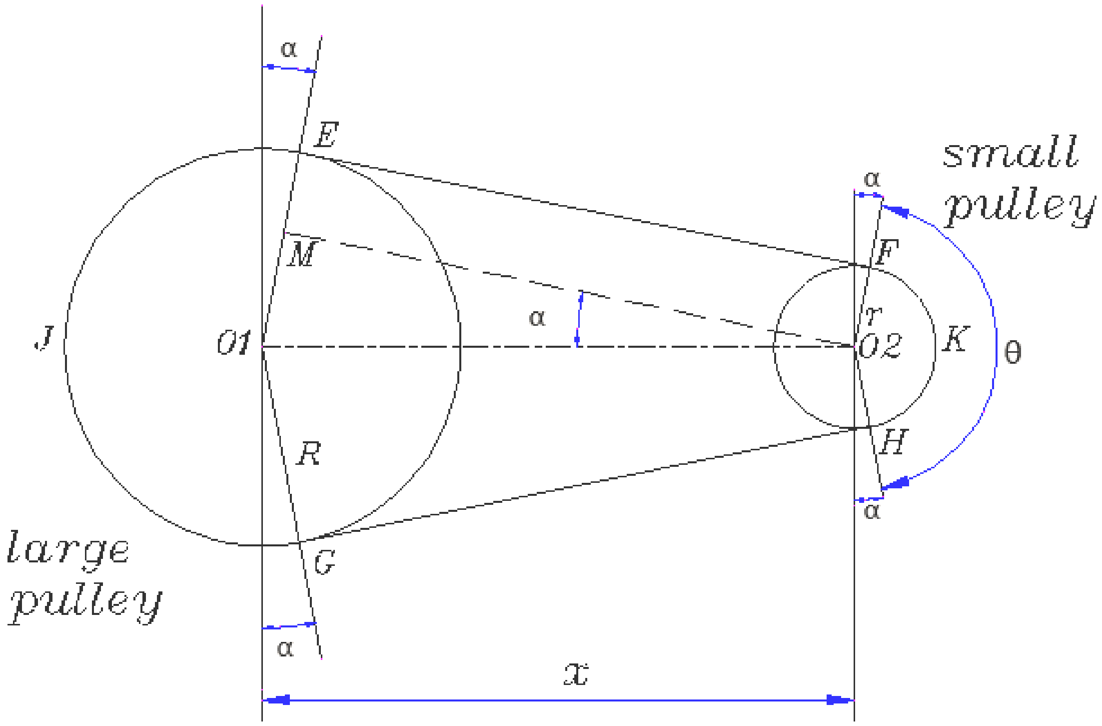
$$V_R = \frac{N_2}{N_1} = \frac{\omega_2}{\omega_1} = \frac{d_1}{d_2}$$

Ⓑ **By account slip**

$$V_R = \frac{N_2}{N_1} = \frac{\omega_2}{\omega_1} = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right)$$

2- Length of belt

Ⓐ***length of an open belt drive:***



Total length of belt is equal to.

$$L = \overset{\frown}{GJE} + EF + \overset{\frown}{FKH} + HG$$

$$L = 2(\overset{\frown}{JE} + EF + \overset{\frown}{FK})$$

from $\triangle\, M O_2 O_1$ *we get*

$$\sin\alpha = \frac{R-r}{x} \Longrightarrow \alpha = \frac{R-r}{x}$$

$$Arc\widehat{JE} = R\left(\frac{\pi}{2}+\alpha\right)$$

$$Arc\widehat{FK} = r\left(\frac{\pi}{2}-\alpha\right)$$

$$EF = \sqrt{x^2-(R-r)^2} = x\sqrt{1-\left(\frac{R-r}{x}\right)^2}$$

Expanding by binomial theory

$$EF = x\left(1-\frac{1}{2}\left(\frac{R-r}{x}\right)^2+.....\right) \approx x-\frac{(R-r)^2}{2\cdot x}$$

$$L = 2\left(R\left(\frac{\pi}{2}+\alpha\right)+x-\frac{(R-r)^2}{2\cdot x}+r\left(\frac{\pi}{2}-\alpha\right)\right)$$

$$L = R\pi+2\cdot\alpha\cdot R+2\cdot x-\frac{(R-r)^2}{x}+r\pi-2\cdot\alpha\cdot r$$

$$L = \pi(R+r)+2\cdot x-\frac{(R-r)^2}{x}+2\cdot\alpha(R-r)$$

$$L = \pi(R+r)+2\cdot x-\frac{(R-r)^2}{x}+2\cdot\frac{(R-r)}{x}\cdot(R-r)$$

$$\boxed{L = \pi(R+r)+2x+\frac{(R-r)^2}{x}}$$

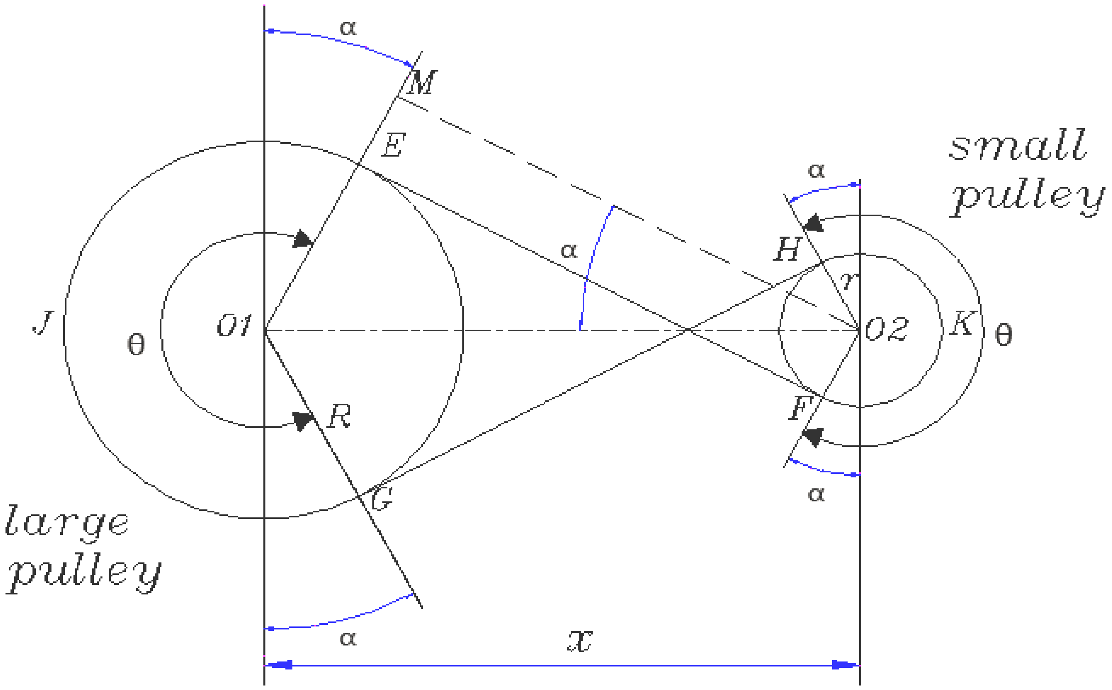
The angle of contact θ

$$\boxed{\theta_{rad} = \frac{\pi}{180}(180-2\alpha)}$$

For small pulley

$$\boxed{\alpha = \sin^{-1}\left(\frac{R-r}{x}\right)}$$

Ⓑ Length of cross belt drive :



By the same way

$$L = \pi(R+r) + 2x + \frac{(R+r)^2}{x}$$

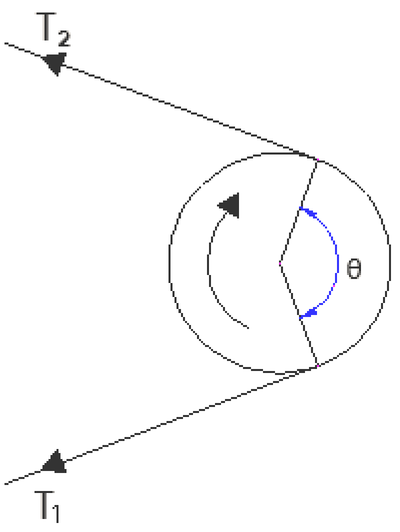
The angle of contact θ

For small pulley

$$\theta_{rad} = \frac{\pi}{180}(180 + 2\alpha)$$

$$\alpha = \sin^{-1}\left(\frac{R+r}{x}\right)$$

3-Ratio of driving tensions :



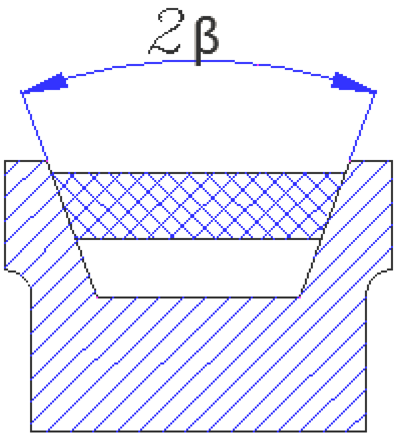
T_1: *tension of tight side. (N)*
T_2: *tension of slack side. (N)*
Ratio of tension forces depend on θ *and* μ

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

For flat belt

$$\frac{T_1}{T_2} = e^{\left(\frac{\mu\theta}{\sin\beta}\right)}$$

For V- belt



Where θ*: angle of contact (radian)*
μ*: coefficient of friction*
2β*: groove angle*

4-Power transmitted by belt:

$$P = (T_1 - T_2) \cdot V$$

Watt

$$P = \frac{(T_1 - T_2) \cdot V}{750}$$

h.p

$$V = \frac{2 \cdot \pi \cdot N_1}{60} \cdot R = \frac{2 \cdot \pi \cdot N_2}{60} \cdot r$$

V : velocity of belt (m/s)

Example (1)

*Two pulleys with diameters **(120 mm)** and **(100 mm)**, centre distance **(300 mm).** Determine the length of belt in case of open and cross belt drive.*

Solution:

$R = 60mm \quad , \quad r = 50mm \quad , \quad x = 300mm$

In case of open belt drive

$$L = \pi(R + r) + 2x + \frac{(R - r)^2}{x}$$

$$L = \pi(60 + 50) + 2 \times 300 + \frac{(60 - 50)^2}{300} = 946mm$$

In case of cross belt drive

$$L = \pi(R + r) + 2x + \frac{(R + r)^2}{x}$$

$$L = \pi(60 + 50) + 2 \times 300 + \frac{(60 + 50)^2}{300} = 986mm$$

Example (2)

*Two pulleys with diameters **(450 mm)** and **(200 mm)**, centre distance **(1.95 m)**. if we use cross belt drive. Determine the length of flat belt, and the contact angle between belt and smaller pulley. Also determine the horse power. If the large pulley turns with speed **(200 rpm)** and the tension force in tight side is **(1000 N),** coefficient of friction **μ=0.25***

Solution:

$R = 0.225m \quad , \quad r = 0.1m \quad , \quad x = 1.95m$

$N_1 = 200.rpm \quad , \quad T_1 = 1000N \quad , \quad \mu = 0.25$

$$L = \pi(R + r) + 2x + \frac{(R + r)^2}{x}$$

$$L = \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975m$$

$$= 4975mm$$

$$\alpha = \sin^{-1}\left(\frac{R+r}{x}\right) = \sin^{-1}\left(\frac{0.225+0.1}{1.95}\right) = 9.6^{o}$$

$$\theta_{rad} = \frac{\pi}{180}(180 + 2\alpha) = 3.477\,rad$$

$$\frac{T_1}{T_2} = e^{\mu\theta} \Longrightarrow \frac{1000}{T_2} = e^{0.25\times3.477}$$

$$T_2 = 419.3N$$

$$V = \frac{2\cdot\pi\cdot N_1}{60}\cdot R = \frac{2\times\pi\times200}{60}\times0.225 = 4.7m/s$$

$$P = \frac{(T_1 - T_2)\cdot V}{750} = \frac{(1000-419.3)\times4.7}{750} = 3.64h.p$$

Example (3)

The diameter of Two pulleys are (0.3 m) and (0.2 m), centre distance (1 m). if we use open belt drive. Determine

1. *The length of V- belt. ($2\beta=60^o$)*
2. *The contact angle between belt and smaller pulley.*
3. *The horse power. If the smaller pulley turns with speed (400rpm) and the tension force in tight side (T_1= 900N), coefficient of friction* ***μ=0.3***
4. *The speed of the larger pulley*

SOLUTION :

$R = 0.15m$, $r = 0.1m$, $x = 1m$

$N_2 = 400.rpm$, $T_1 = 900N$, $\mu = 0.3$

1-

$$L = \pi(R+r) + 2x + \frac{(R-r)^2}{x}$$

$$L = \pi(0.15+0.1) + 2\times1 + \frac{(0.15-0.1)^2}{1} = 2.79m$$

$$= 2790mm$$

2-

$$\alpha = \sin^{-1}\left(\frac{R-r}{x}\right) = \sin^{-1}\left(\frac{0.15-0.1}{1}\right) = 2.87^o$$

$$\theta_{rad} = \frac{\pi}{180}(180-2\alpha) = 3.04rad$$

3-

$$\frac{T_1}{T_2} = e^{\frac{\mu\theta}{\sin\beta}} \Longrightarrow \frac{900}{T_2} = e^{\frac{0.3\times 3.04}{\sin 30}}$$

$$T_2 = 145.2N$$

$$V = \frac{2\cdot\pi\cdot N_2}{60}\cdot r = \frac{2\times\pi\times 400}{60}\times 0.1 = 4.2m/s$$

$$P = \frac{(T_1-T_2)\cdot V}{750} = \frac{(900-145.2)\times 4.2}{750} = 4.23h.p$$

4-

$$V = \frac{2\cdot\pi\cdot N_1}{60}\cdot R = \frac{2\cdot\pi\cdot N_2}{60}\cdot r$$

$$N_1\cdot R = N_2\cdot r$$

$$N_1\times 0.15 = 400\times 0.1$$

$$N_1 = 266.7rpm$$

Clutch

Function: it is a machine part which translates or transmits motion from one part to another.

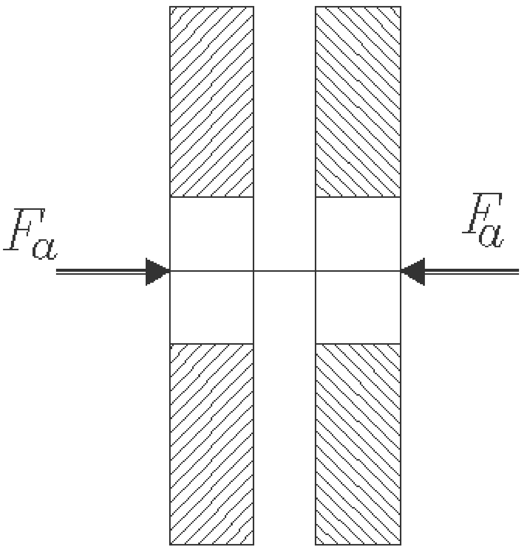
Types of clutches:

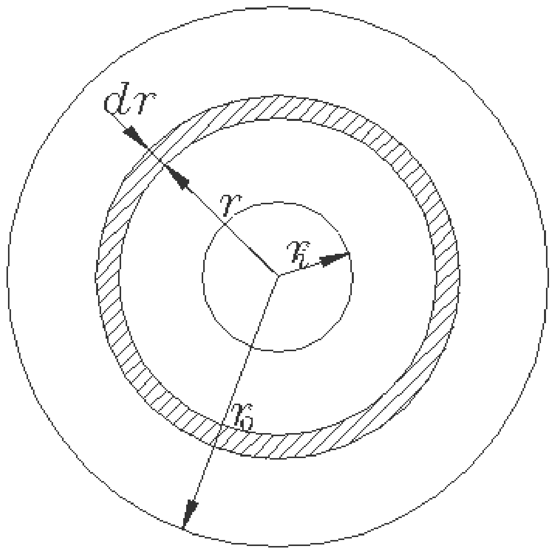
I) friction clutch: Divide into:
- *Disc clutch*
  - *single disc*
  - *Multiple discs*
- *Cone clutch*

II) Hydraulic clutch
III) Electro –magnetic clutch
IV) Automatic clutch

I) Friction clutch

Consider two flat surfaces, maintained in contact by axial thrust (F_a)





Let
r_o *:outer radius of clutch (m)*
r_i*: inner radius of clutch (m)*
T_f *: torque transmitted by friction (N.m)*
P: axial pressure (N/m^2)
F_a*: axial force (thrush force) (N)*
μ*: coefficient of friction.*

The force acting on element = dF_a

$$dF_a = p \cdot (2\pi \cdot r \cdot dr)$$

$$dF_f = \mu \cdot dF_a$$

$$dT_f = dF_f \cdot r$$

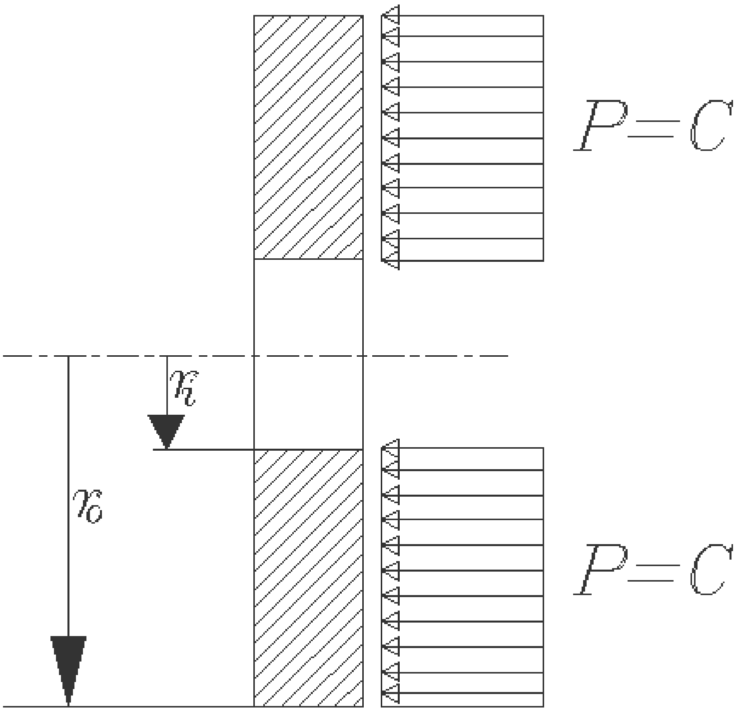
$$dT_f = p \cdot \mu \cdot 2\pi \cdot r^2 \cdot dr$$

The design of friction clutch must be done on the base of

1- Uniform pressure: (new clutch)

$$P = constant = P_{max}$$

$$dF_a = p \cdot (2\pi \cdot r \cdot dr)$$



$$F_a = \int_{r_i}^{r_o} p \cdot (2\pi \cdot r \cdot dr) = p \cdot \left[2\pi \cdot \frac{r^2}{2} \right]_{r_i}^{r_o}$$

$$\boxed{F_a = \pi \cdot p_{\max} \cdot \left(r_o^2 - r_i^2 \right)}$$

$$dT_f = p \cdot \mu \cdot 2\pi \cdot r^2 \cdot dr$$

$$T_f = p_{\max} \cdot \mu \int_{r_i}^{r_o} 2 \cdot \pi \cdot r^2 \cdot dr = p_{\max} \cdot \mu \cdot \left[\frac{2}{3} \cdot \pi \cdot r^3 \right]_{r_i}^{r_o}$$

$$T_f = p_{\max} \cdot \mu \cdot \frac{2}{3} \cdot \pi \cdot \left(r_o^3 - r_i^3 \right)$$

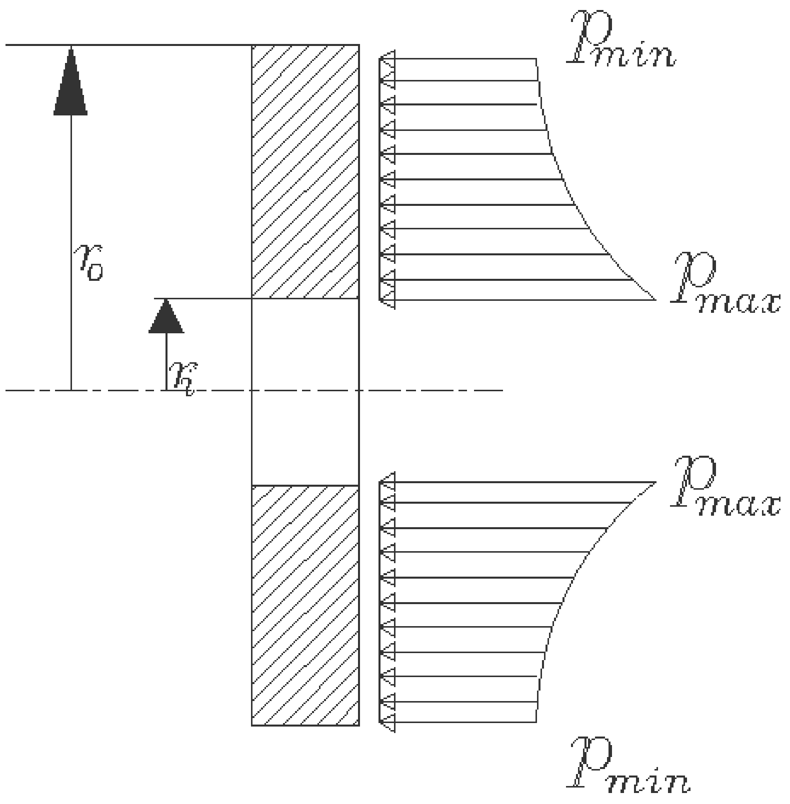
$$T_f = \frac{2}{3} \cdot \left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \right) \cdot \mu \cdot F_a$$

2-Uniform wear: (old clutch)

$$p \cdot r = constsnt$$

$$p_{\max} \cdot r_i = C$$

$$p_{\min} \cdot r_o = C$$



$$dF_a = p \cdot (2\pi \cdot r \cdot dr)$$

$$F_a = 2\pi \int_{r_i}^{r_o} p \cdot r \cdot dr = 2 \cdot \pi \cdot C \int_{r_i}^{r_o} dr = 2 \cdot \pi \cdot C \cdot (r_o - r_i)$$

$$F_a = 2 \cdot \pi \cdot C \cdot (r_o - r_i)$$

$$dT_f = p \cdot \mu \cdot 2\pi \cdot r^2 \cdot dr$$

$$T_f = 2\pi \cdot \mu \int_{r_i}^{r_o} p \cdot r^2 \cdot dr = 2\pi \cdot \mu \int_{r_i}^{r_o} p \cdot r \cdot r \cdot dr$$

$$T_f = 2\pi \cdot \mu \cdot C \cdot \int_{r_i}^{r_o} r \cdot dr = 2\pi \cdot \mu \cdot C \cdot \left[\frac{r_o^2}{2} - \frac{r_i^2}{2}\right]$$

$$T_f = \pi \cdot \mu \cdot C \cdot \left(r_o^2 - r_i^2\right)$$

$$T_f = \mu \cdot F_a \cdot \left(\frac{r_o + r_i}{2}\right)$$

Consideration must be taken in design of friction clutch:

1-for multi-disc clutch let **n** *be the number of pairs of contact surfaces.*

$$T_f = n \cdot \mu \cdot F_a \cdot R_f$$

$$R_f = \frac{2}{3} \cdot \left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right) \qquad \text{For uniform pressure}$$

$$R_f = \frac{r_o + r_i}{2} \qquad \text{For uniform wear}$$

R_f : friction radius

2- if there are n_1: number of discs on the driver
n_2: number of discs on the driven
Then the number of pairs of contact surface

$$n = n_1 + n_2 - 1$$

3-Recommanded the ratio

$$0.6 < \frac{r_i}{r_o} < 0.8$$

4- Friction torque must be more than engine torque

$$T_f = \beta \cdot T_{engine}$$

β *:engagement factor*

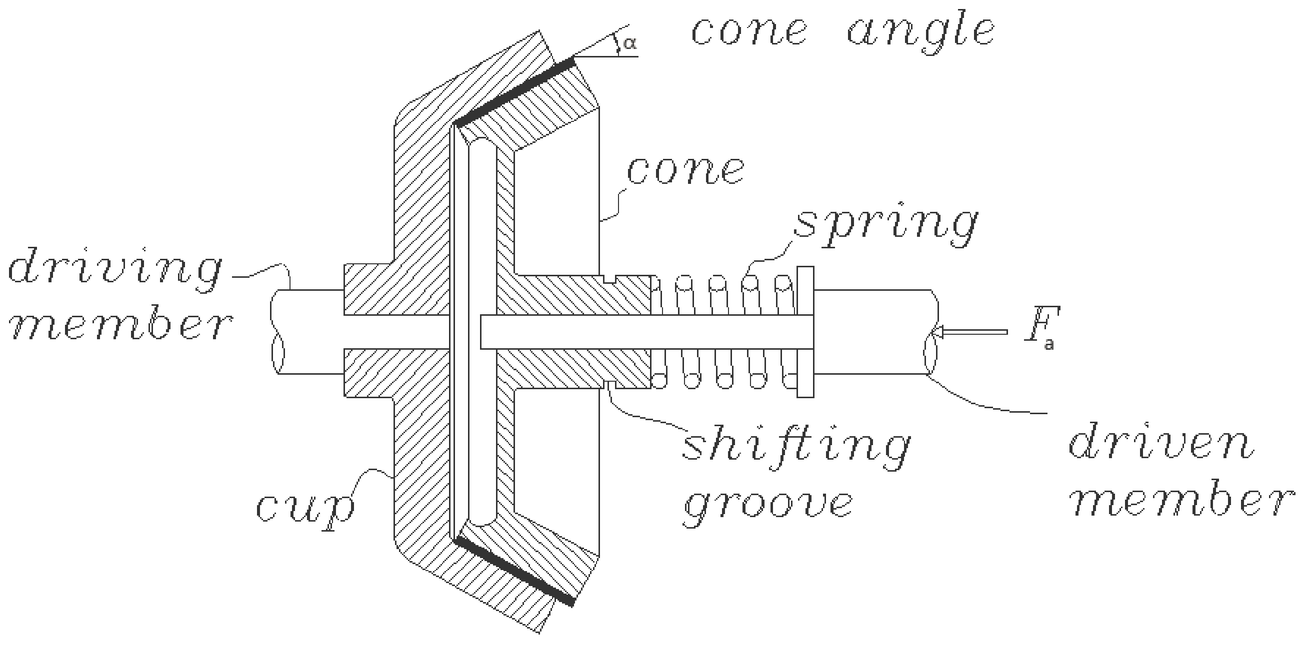
$\beta = 1.25 \rightarrow 1.5$ For machine

$\beta = 1.2 \rightarrow 1.5$ For car

$\beta = 2 \rightarrow 2.5$ For tractors

Cone Clutch :

It is used to transmit high torque because of large friction area. specially used with low peripheral speed.



Let

α *:cone face angle*

$$\alpha = 8^o \rightarrow 12.5^o$$

F_a*: axial force (spring force) (N)*

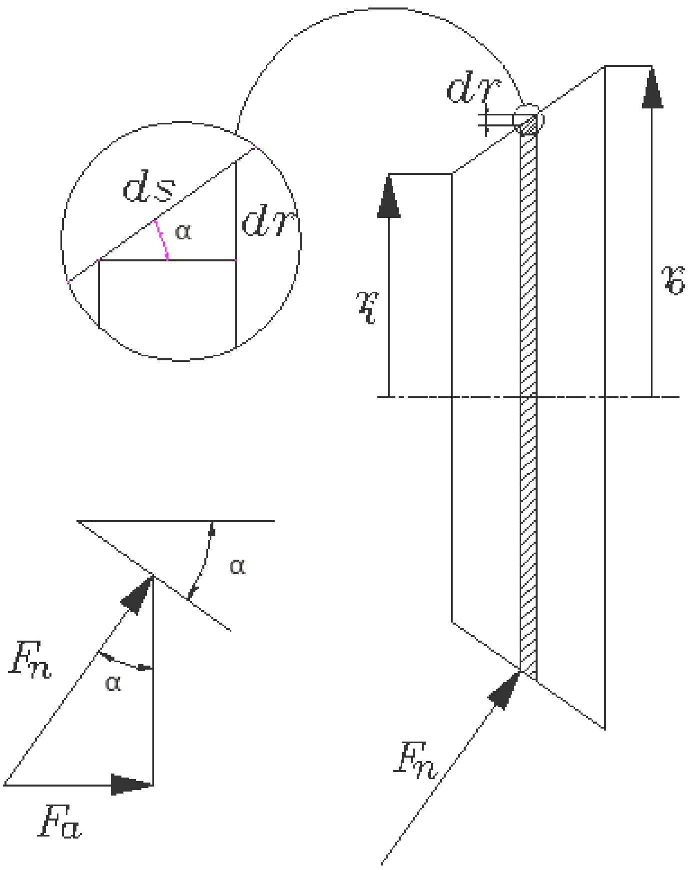
P_n*: normal pressure (N/m^2)*

F_n*: normal Force (N)*

$$\sin\alpha = \frac{F_a}{F_n}$$

$$F_a = F_n \cdot \sin\alpha$$

Element of area $$dA = 2\pi \cdot r \cdot ds = 2\pi \cdot r \cdot \frac{dr}{\sin\alpha}$$



$$dF_n = p_n \cdot \left(2\pi \cdot r \cdot \frac{dr}{\sin\alpha} \right)$$

$$dF_a = p_n \cdot \left(2\pi \cdot r \cdot \frac{dr}{\sin\alpha} \right) \cdot \sin\alpha$$

$$dF_a = 2\pi \cdot p_n \cdot r \cdot dr$$

$$dT_f = dF_{n} \cdot \mu \cdot r = 2\pi \cdot \mu \cdot p_n \cdot r^2 \cdot \frac{dr}{\sin\alpha}$$

1-uniform pressure $p_n = C$

$$dF_a = 2\pi \cdot p_n \cdot r \cdot dr$$

$$F_a = 2\pi \cdot p_n \int_{r_i}^{r_o} r \cdot dr = 2\pi \cdot p_n \cdot \left[\frac{r^2}{2} \right]_{r_i}^{r_o}$$

$$F_a = \pi \cdot p_n \cdot \left(r_o^2 - r_i^2 \right)$$

$$dT_f = 2\pi \cdot \mu \cdot p_n \cdot r^2 \cdot \frac{dr}{\sin\alpha}$$

$$T_f = 2\pi \cdot \mu \cdot p_n \cdot \frac{1}{\sin\alpha} \int_{r_i}^{r_o} r^2 \cdot dr = 2\pi \cdot \mu \cdot p_n \cdot \frac{1}{\sin\alpha} \left[\frac{r^3}{3} \right]_{r_i}^{r_o}$$

$$T_f = \frac{\pi \cdot p_n \cdot \mu}{\sin\alpha} \cdot \frac{2}{3} \cdot \left(r_o^3 - r_i^3 \right)$$

$$T_f = \frac{2}{3} \cdot \left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \right) \cdot \frac{\mu \cdot F_a}{\sin\alpha}$$

2-uniform wear $p_n \cdot r = C$

$$dF_a = 2\pi \cdot p_n \cdot r \cdot dr$$

$$F_a = 2\pi \int_{r_i}^{r_o} p_n \cdot r \cdot dr = 2 \cdot \pi \cdot C \int_{r_i}^{r_o} dr = 2 \cdot \pi \cdot C \cdot \left(r_o - r_i \right)$$

$$F_a = 2 \cdot \pi \cdot C \cdot \left(r_o - r_i \right)$$

$$dT_f = 2\pi \cdot \mu \cdot p_n \cdot r^2 \cdot \frac{dr}{\sin\alpha}$$

$$T_f = 2\pi \cdot \mu \cdot \frac{p_n \cdot r}{\sin\alpha} \int_{r_i}^{r_o} r \cdot dr$$

$$T_f = \frac{2\pi \cdot \mu \cdot C}{\sin\alpha} \cdot \left[\frac{r_o^2}{2} - \frac{r_i^2}{2} \right]$$

$$T_f = \frac{\pi \cdot \mu \cdot C}{\sin\alpha} \cdot \left(r_o^2 - r_i^2 \right)$$

$$T_f = \frac{\mu \cdot F_a}{\sin\alpha} \cdot \left(\frac{r_o + r_i}{2} \right)$$

Example (1)

*A multi-disc clutch employ **3** steel and **2** bronze disc having outer diameter **(30 cm)** and inner diameter **(20 cm)**. Find the axial force and the power transmitted if the normal pressure is **(P = 1.3 kg /cm²)** and **(N=750 rpm).**take uniform wear **(μ =0.22)***

Solution :

$r_o = 15cm = 0.15m \qquad , \qquad r_i = 10cm = 0.1m$

$$p_{max} = 1.3\frac{kg}{cm^2} = 13\frac{N}{cm^2} = 13\frac{N}{\frac{m^2}{10^4}} = 13\times10^4\frac{N}{m^2} \qquad , \qquad N = 750rpm$$

Uniform wear $\qquad , \qquad \mu = 0.22$

$$F_a = 2\cdot\pi\cdot C\cdot(r_o - r_i)$$
$$= 2\cdot\pi\cdot(p_{max}\cdot r_i)\cdot(r_o - r_i)$$
$$F_a = 2\times\pi\times(13\times10^4\times0.1)\times(0.15-0.1) = 4084N$$
$$n = n_1 + n_2 - 1$$
$$n = 3+2-1 = 4$$
$$T_f = n\cdot\mu\cdot F_a\cdot R_f \qquad \because R_f = \frac{r_o + r_i}{2}$$
$$T_f = 4\times0.22\times4084\times\left(\frac{0.15+0.1}{2}\right) = 449.24N\cdot m$$

Power transmitted $= P = T_f \cdot \omega$